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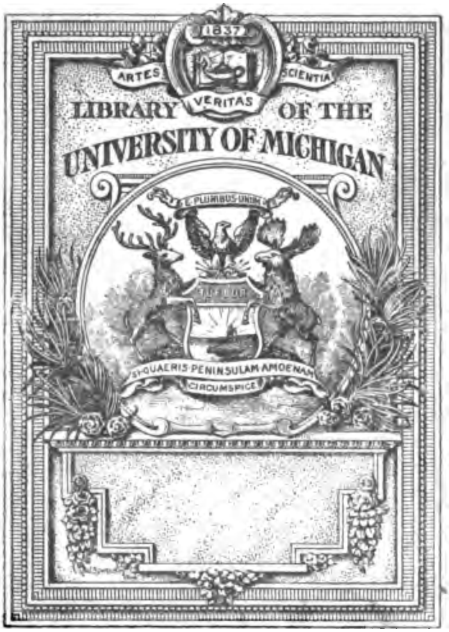
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EXERCISES

IN

ALGEBRA,

BEING

SYSTEMATICALLY ARRANGED AND ADAPTED TO
THE GRADUAL PROGRESS OF YOUNG PUPILS
IN ACADEMIES AND SCHOOLS,

WITH

A KEY

FOR

THE USE OF THE TEACHER.

oor. p. lv

BY FRANCIS J. GRUND,

Author of an "Elementary Treatise on Plane and Solid Geometry,"
"Elementary Treatise on Natural Philosophy," "Elements
of Chemistry," "Popular Lessons in Astronomy,"
translator of "Meier Hirsch's Problems," etc.

BOSTON:
CARTER, HENDEE AND CO.
1833.

Entered according to act of Congress, in the year 1833,
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in the clerk's office of the District Court of Massachusetts.

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PREFACE.

The want of a suitable book to exercise the learner in Algebra, in the rules which are essential to a proper understanding of the other branches of Mathematics, and which shall at the same time enable him to solve with facility such questions as occur most frequently in the various transactions of common life, must have been felt long ago, by every experienced teacher of Mathematics. To satisfy teachers in this respect, and in order to afford to those who are studying without a teacher the means of practice in this department of mathematical science, the author, advised by friends, who are much better judges than himself, offered to the public a collection of Algebraic Problems and Formulæ, translated from the celebrated collection of Meier Hirsch. But this work, although excellent of its kind, is not sufficiently elementary for young beginners, and it is on this account the author submits to the consideration of teachers generally, the present collection of Algebraic Exercises. They contain principally such sums and problems as have an immediate bearing on what is wanted for admission into most American Colleges.

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They are, it is believed, systematically arranged, and follow each other in such regular gradations, as will enable the learner, with very little assistance from the teacher, to work his own way. They do not go further than Equations of the second Degree ; but contain, it is believed, an unusual variety of examples in the preceding rules, and hints to the pupils for solving the more difficult ones among them. No answers are given in the book; because *a Key for the use of the teacher* accompanies the work, and contains besides the answers, the statement of the equations of the most difficult problems.

Those that wish to proceed farther in Algebra, we refer to the problems of Meier Hirsch, published by Carter, Hendee & Co. in Boston.

If the present collection of Algebraic problems should prove acceptable, the author intends to have it followed by Exercises in Geometry and Trigonometry, which will be executed on the same plan, and be a companion to this work.

A collection of Arithmetical Exercises, is already in press, and will be published in a few weeks.

Boston, September, 1833.

ERRATA.

Page 8, 1st line from above, read 10 $b\ c$ instead of 1 $b\ c$

Page 16, 5th line from above, read 2^{-3} instead of 2^{-2}

Page 17, 3d line from above, read 19.6^4 instead of 196^4

Page 21, problem 43, read $\times$ instead of $+$; *ibid.* problem 49, read $\times$ instead of $+$

Page 25, 1st line, read, Division of Powers, instead of Division of Compound; the † belongs to exercise 19, and not to exercise 21.

Page 26, exercise 14, read a^3 instead of a^2

Page 43, exercise 11, read 37636 instead of 37830; *ibid.* exercise 40, read $\frac{256}{361}$ for $\frac{256}{261}$

Page 54, exercise 2, read

$$\left.\begin{array}{l}\text{If}\quad \sqrt{8} = \sqrt{4}.\sqrt{2}\\ \text{and}\ \sqrt{18} = \sqrt{9}.\sqrt{2}\end{array}\right\}\text{ instead of }\left\{\begin{array}{l}2\sqrt{8} = \sqrt{4}.\sqrt{2}\\ 3\sqrt{18} = \sqrt{9}.\sqrt{2}\end{array}\right.$$

Page 61, exercise 9, read $a^{\frac{1}{2}} \times a^{\frac{1}{3}}$ instead of $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$

Page 64, exercise 16, read $(a^{\frac{5}{2}})^{\frac{1}{4}}$ instead of $(a^{\frac{1}{4}})^{\frac{1}{3}}$

Page 70, exercise 45, read, divided by 5 instead of divide by 7.

Page 73, exercise 6, read $\sqrt[4]{1-x} = (1-x)^{\frac{1}{4}}$ instead of $\sqrt[4]{1+x} = (1+x)^{\frac{1}{4}}$

Page 86, exercise 35, 3d line, read $\frac{3}{4}x + 2y + 3\frac{1}{2}z$ instead of $\frac{3}{4}x + \frac{1}{2}y + 3\frac{1}{2}z$

Page 88, exercise 11, read $76\frac{7}{8}$ instead of $73\frac{1}{2}$; *ibid.* exercise 18, read 6384 instead of 638.

Page 92, problem 9, read $\frac{1}{8}$ for coffee instead of $\frac{1}{3}$

Page 93, problem 11, read $\frac{1}{8}$ in my youth instead of $\frac{7}{8}$; *ibid.* problem 16, read $\frac{1}{4}$ instead of $\frac{1}{3}$

Page 95, 2d line from above, read $\frac{23}{20}$ instead of $\frac{25}{20}$

Page 96, problem 12, read 2*s.* 6*d.* instead of 2*s.* 9*d.*

Page 100, problem 7, read 1*s.* 4*d.* instead of 1*s.* 3*d.*

Page 103, problem 19, read, A is as old as D, B, and C, taken together instead of D is as old as A, B, and C, &c.

Page 104, problem 3, read 259 instead of 250.

Page 107, problem 4, read $\frac{1}{8}$ instead of $\frac{1}{3}$

Page 120, problem 29, read *and you* instead of *as you.*

Page 142, in the remark to problem 144, read from $\frac{2}{5}x - 70$ instead of $\frac{2}{5}x - 75$.

Page 152, problem 3, read, if he puts \$10 *into the* instead of \$10 *out of.*

Page 166, problem 52, 9th line from above, read \$3,228 instead of \$3,225.

Page 177, problem 23, read, as the 12th part of the purchase money instead of the $12\frac{1}{2}$ part of the purchase money.

Page 187, in problem 1, read 3 per cent instead of 4 per cent, and 1.03 instead of 1.03.

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EXERCISES IN ALGEBRA.

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PROBLEMS IN ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

A. Addition.

a. ADDITION OF SIMPLE QUANTITIES.

$$\begin{array}{rrrrrrrrrrrr}
1) & a & 2) & 7a & 3) & 4b & 4) & 5f & 5) & a & 6) & 7a \\
 & \underline{a} & & \underline{7a} & & \underline{2b} & & \underline{f} & & b & & \underline{10b}
\end{array}$$

$$\begin{array}{rrrrrrrrrrrr}
7) & a & 8) & 17a & 9) & -3a & 10) & a & 11) & 8a & 12) & -7a \\
 & \underline{-a} & & \underline{-6a} & & \underline{2a} & & \underline{-b} & & \underline{-5b} & & \underline{b}
\end{array}$$

$$\begin{array}{rrrrrrrrrr}
13) & -14a & 14) & -11a & 15) & 8a & 16) & 3a & 17) & 6f \\
 & \underline{-3a} & & \underline{-a} & & \underline{3b} & & -5a & & -9f \\
 & & & & & & & \underline{8a} & & -13f \\
 & & & & & & & & & \underline{f}
\end{array}$$

$$\begin{array}{rrrrrrrr}
18) & -12b & 19) & -7c & 20) & 5d & 21) & 8e \\
 & -4b & & -5c & & 7d & & -4e \\
 & \underline{13b} & & \underline{-3c} & & d & & 7e \\
 & & & & & \underline{-8d} & & \underline{-11e}
\end{array}$$

22) $$\begin{array}{r} 3g \\ 2g \\ -5g \\ 7g \\ -7g \\ \hline \end{array}$$

23) $$\begin{array}{r} 3a \\ 4a \\ 7a \\ -3a \\ -2a \\ -a \\ \hline \end{array}$$

24) $$\begin{array}{r} 7b \\ 3b \\ b \\ -4b \\ -3b \\ -2b \\ -b \\ \hline \end{array}$$

25) $a + 3a + 2a =$ what?

26) $2a + 4b - 7c + 3a + 7c =$ what?

27) $3a + b + 5a + 9b - 8a =$ what?

28) $4a - 5b + 4c - 5a + 4b =$ what?

29) $3a + 4b - 7d + a - 2b + 3d =$ what?

30) $11a - 4b - c + d - 8a =$ what?

31) $a - 3b + 4b + 4a - d =$ what?

32) $14a - 14b - 3a - 4b - c + 4a - b =$ what?

33) $9a - 3b - 2a - d - 3b =$ what?

34) $7a - 2b - 4a - 3a + 2a =$ what?

35) $9a + 3b - 2a - a + 2b =$ what?

36) $10a + 14b + 3b + 9a + b =$ what?

---

b. ADDITION OF COMPOUND QUANTITIES.

1) $$\begin{array}{r} 7a - 5c + 3b \\ -2a - 3c - 7b \\ \hline \end{array}$$

2) $$\begin{array}{r} 5a + 4b - 3c \\ 3a - 12b + 7c \\ \hline \end{array}$$

3) $$\begin{array}{r} 4a - 7b - 3c + 8 \\ 3a + 12b - 10c + 2 \\ \hline \end{array}$$

4) $$\begin{array}{r} 12h - 3c - 7f + 3g \\ -3h + 8c - 2f - 9g \\ \hline \end{array}$$

5) $$\begin{array}{r} 16a - 5b + 15c - 9d \\ 3a + 18b - 5c - 7d \\ -7a - 2b + c - d \\ \hline \end{array}$$

6) $$\begin{array}{rrrr} 8a & +b & & \\ 2a & -b & +c & \\ 7a & +5b & & +2d \\ & -6b & -3c & +3d \\ \hline \end{array}$$

7) $$\begin{array}{rr} -7f & +3a \\ 4f & -2a \\ 3f & -3a \\ & +2a \\ \hline \end{array}$$

8) $$\begin{array}{rrrr} 7a & -6y & +5z & \\ -x & -3y & & -8 \\ -x & +y & -3z & -1 \\ -2x & +3y & +3z & -1 \\ \hline \end{array}$$

9) $$\begin{array}{rrrr} 7a & +b & & \\ 2a & -b & +c & \\ -3a & +5b & & +2d \\ & -6b & -3c & +3d \\ -5a & & +7c & -2d \\ \hline \end{array}$$

10) $$\begin{array}{rrrrrr} 3a & +4b & -2c & +d & & \\ 2a & -2b & -5c & -2d & +g & \\ & -b & +4c & -3d & & -f \\ \hline \end{array}$$

11) $$\begin{array}{rrrrrr} 13a & -4b & -c & & & \\ 4a & +2b & -3c & -d & & \\ 5a & +b & +4c & -2d & -e & \\ 2a & +b & +5c & -3d & +2e & -f \\ \hline \end{array}$$

12) $$\begin{array}{rrrrr} 6a & & & & \\ 7a & +b & & & \\ 3a & -2b & +3c & & \\ -2a & -b & -2c & +d & \\ -3a & -4b & -c & -2d & +e \\ \hline \end{array}$$

B. Subtraction.

a. SUBTRACTION OF SIMPLE QUANTITIES.

1) From a
Subtract a

2) From $7a$
Subtract $3a$

3) From $10f$
Subtract $10f$

4) From $5d$
Subtract $7d$

5) From $7a$
Subtract $5b$

6) From a
Subtract $-a$

7) From $8a$
Subtract $-a$

8) From $6a$
Subtract $-5a$

9) From a
Subtract $-a$

10) From a
Subtract $-b$

11) From $3a$
Subtract $-2b$

12) From $-9a$
Subtract $3a$

13) From $-7a$
Subtract $-3a$

14) From $-7a$
Subtract $-7a$

15) From $-19a$
Subtract $-20a$

16) From $-6a$
Subtract $-5a$

17) From $-3a$
Subtract $-5b$

18) From -13
Subtract 3

19) From -8
Subtract -17

20) From 12
Subtract -7

21) From -13
Subtract -8

22) From -8
Subtract 8

23) From -8
Subtract -8

24) From 8
Subtract -8

b. SUBTRACTION OF COMPOUND QUANTITIES.

1) From $3a - 5b$
Subtract $\underline{2a - 3b}$

2) From $4a - 7b$
Subtract $\underline{3a + 5b}$

3) From $5d + 7f$
Subtract $\underline{2d - 3f}$

4) From $3a - 2b + 6$
Subtract $\underline{2a - 7b - 3}$

5) From $13a - 2b + 9c - 3d$
Subtract $\underline{8a - 6b + 9c - 10d}$

6) From $-7f + 14b - 9d$
Subtract $\underline{-7f - 9b - 9d}$

7) From $-7f + 3m - 8x$
Subtract $\underline{-6f - 5m - 2x + 3d}$

8) From $2a - e - h - l$
Subtract $\underline{9a - 3e + 4h - l - c}$

9) From $-a - 5b + 7c - d$
Subtract $\underline{4b - 3c - 2d + 3k}$

10) From $3h - 2k + l$
Subtract $\underline{9h - 8k - 7 - l}$

11) From $3h - 2k$
Subtract $\underline{9l - 7 - 8k}$

12) From $4h - a + k$
Subtract $\underline{a - 4k + h}$

13) From $-3a + b - 8c + 7e - 5f$
Subtract $\underline{2a - 9c + 8e - 7}$

14) From $a + b + 7e$
Subtract $\underline{k - a + b + 7e - k}$

15) From $5b - 3a + 203c + 5$
Subtract $-2b - 8a + 67c + 7$

16) From $3a - 17b - 10b + 13a - 2a$
Subtract $6a - 8a - b - 2a + 3d + 9a$

17) From $5c + 3$
Subtract $2c - 9 - 7c$

18) From $8a - 5b - 3c - 7d + 5e - 8f + 3g$
Subtract $-2k + 3c - 5b + 2\,4 - de - 7f + 9g - 5k$

19) $32a + 3b - (5a + 17b) =$ what ?
20) $13a - (5c + 3f - 7a - 5x) =$ what ?
21) $-8a + 5b - 3c - (7a - 3b - 2c) =$ what ?
22) $3a - 5c + 3d - (7a - 5d + 8c - 2e) =$ what ?
23) $37a - 5f - (3a - 2b) - (6a - 4b + h) =$ what ?
24) $a + b - (2a - 3b) =$ what ?
25) $4z - (37 - x - z) - (2x - 27 - 2z) =$ what ?
26) $a + b - (2a - 3b) - (5a + 7b) -$
$(-13a + 2b) =$ what ?
27) $a + b - (3a - 2b) - (4a + 3b) - (a + b) =$ what ?
28) $a + b - (a - b) =$ what ?
29) $a + b - (a - b) - (a - b) =$ what ?
30) $a + b - (a - b) - (a - b) - (a - b) =$ what ?

C. *Multiplication.*

a. MULTIPLICATION OF SIMPLE QUANTITIES.

1) $a \times b =$ what ?
2) $b \times a =$ what ?
3) $a \,.\, b =$ what ?
4) $b \,.\, a =$ what ?

5) $-a \times b =$ what? 6) $a \times -b =$ what?
7) $-a \times -b =$ what? 8) $6a \times 7b =$ what?
9) $-3a \times 14c =$ what? 10) $-7a \times 10b =$ what?
11) $a \times -7b =$ what? 12) $-6a \times -11x =$ what?
13) $a \times b \times c =$ what? 14) $a \, . \, b \, . \, c =$ what?
15) $ab \times cde =$ what?
16) $-5abc \times -7ade =$ what?
17) $-5bd \times 9bbdxy =$ what?
18) $17ace \times 5 =$ what?
19) $2dfg \times 7ab \times 3 =$ what?
20) $4bf \times 3ab \times cde =$ what?
21) $3ab \times 2cd \times dfg =$ what?
22) $2ab \times 3cd \times 4ef \times 5gh =$ what?
23) $-a \times -a \times -a \times -a =$ what?
24) $-a \times a \times -a \times -a =$ what?

---

b. MULTIPLICATION OF COMPOUND QUANTITIES.

1) Multiply $3a + 2b$
by $3c$

2) Multiply $3a + 3b + 5c$
by $2d$

3) Multiply $3a + 2b - 5d + e$
by $7f$

4) Multiply $4a + 5b - d$
by $4e$

5) Multiply $6a + 3b - 5f$
by $5g$

6) Multiply $-2b + 3g - c$
by $-8h$

7) Multiply $7ad - 1bc$
by $10bd$

8) Multiply $5ad + 15bc - 16acf$
by $6ab$

9) Multiply $a + b$
by $c + d$

10) Multiply $a + b$
by $a + b$

11) Multiply $2a + 3b$
by $c + d$

12) Multiply $2a + 3b$
by $4c + 5d$

13) Multiply $a + b - c$
by $d - e$

14) Multiply $2a - 3b - 7c$
by $5f - 6g$

15) Multiply $2a - 3b - 8c - d$
by $7f - 2g - h$

16) Multiply $2a - 3b - 8c - d + g$
by $4f - g + h$

17) Multiply $7l - 2m - 9$
by $3l - 11m$

18) Multiply $2a + 5b + 3c$
by $3a + 10b - 5f$

19) Multiply $3a + 2b + 3c - 5e$
by $3a + 10b + f$

20) $(a + b) \times (a - b) =$ what?

21) $(2a + 5b) \times (3a - 4b) =$ what?

22) $(a - b) \times (a - b) =$ what?

23) $(2a - 5b) \times (3a - 7b) =$ what?

24) $(5ab + 3ac - 4bc) \times (7ab - 8ac) =$ what?

25) $(13bcd + 20bce - 10bde) \times (4bc + 3bd - 12e) =$ what?

26) $(5aa - 3ab + 7bb) \times (3a - b) =$ what?

27) $(a + b + c) \times (a + b - c) =$ what?

28) $(2a + 3b + 4c) \times (2a + 4b - 5c)$

29) $(3aaa + 5abb + 4bbb) \times (4aaa - 5abbb + 4bbb) =$ what?

30) $(3a - b + 2c - 3d) + (17a - 2b + 12c)$

31) $(a + b + c + d) \times (a + b - c - d) =$ what?

32) $(2a + 3b + 4c - 5d) \times (2a + 3b + 4c - 5d) =$ what?

33) $(-2a + 3b - ec) \times (-3f - 7a + cc) =$ what?

34) $(3m - 5n - pp) \times (m - 2n + 6pp) =$ what?

35) $(3mm - nn - pp) \times (mm - nn + pp) =$ what?

36) $(2mm - nn - pp) \times (mm - nn - pp) =$ what?

Division.

a. DIVISION OF SIMPLE QUANTITIES.

1) $a : a =$ what?

2) $aa : a =$ what?

3) $aaa : a =$ what?

4) $aaa : aa =$ what?

5) $aaaa : aa =$ what?

6) $aaaa : aaa =$ what?

7) $a : b =$ what?

8) $-a : b =$ what?

9) $a : -b =$ what?

10) $-a : -b =$ what?

11) $ab : -a =$ what? 12) $-ab : -b =$ what?
13) $6ab : 2a =$ what? 14) $12abc : 2c =$ what?
15) $15abcd : 3bc =$ what? 16) $5a : 3b =$ what?
17) $-16a : 8b =$ what? 18) $12a : -4b =$ what?
19) $-14a : -4b =$ what? 20) $abc : a =$ what?
21) $abc : ad =$ what?
22) $8fmn : -2fgm =$ what?
23) $-12abcde : 8acd =$ what?
24) $6abde : -2bf =$ what?
25) Divide $27aaabbcfg$ by $-18abcghk$.
26) Divide $25aaabbbcffg$ by $-5aabbc.k$

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b. DIVISION OF COMPOUND QUANTITIES.

1) Divide $14ac - 4ade - 6f + 6d$, by 2.
2) Divide $14ac - 4ade - 6af + 8ad$, by $2a$.
3) Divide $3ac - 2ade - f$, by $2a$.
4) Divide $18acf - 6bdef - 2ad$, by $3adf$.
5) Divide $8aa - 6ab + 4c + 1$, by $-2a$.
6) Divide $12acfg - 4affg + 3fggh$, by $4abfg$.
7) Divide $12acfg - 4affg + 3fggh$, by $4aabbfg$
8) Divide $ab - ac$, by $b - c$.
9) Divide $4ab - 8ac$, by $2b - 4c$.
10) Divide $ac - bc + ad - bd$, by $a - b$.
11) Divide $6ac - 4bc + 6ad - 4bd$, by $3a - 2b$.
12) Divide $4aa + 6ab - 4ax + 9bx$
$- 15xx$, by $2a + 3x$.
13) Divide $aa + 2ab + bb$, by $a + b$.
14) Divide $4xxx + 4xx - 29x + 21$, by $2x - 3$.

15) Divide $14af - 21bf + 7cf + 6ag - 9bg + 3cg$, by $7f + 3g$.

16) $(aa + ab + 2ac - 2bb + 7bc - 3cc) : (a + 2b - c) =$ what?

17) $(aa + ab + 2ac - 2bb + 7bc - 3cc) : (a - b + 3c) =$ what?

18) $(4aaaa - 9aaa - 9aa - 5aab + bb) : (aa - 3a - b) =$ what?

19) $$\left\{\begin{matrix} 12aa + 26ab - 36ac + 18ad - 10bb \\ + 29bc - 6bd - 21cc + 9cd \end{matrix}\right\} : (6a - 2b + 3c) = \text{what?}$$

20) $$\left\{\begin{matrix} 119cc - 200cd + 408ee - 113cf - 39dd \\ - 72de + 37df - 96ef + 20ff \end{matrix}\right\} : (17c + 3d - 4f) = \text{what?}$$

21) $(30aab - 6aac + 75abb - 15abc) : (15ab - 3ac) =$ what?

22) $(36aab - 63abb + 20bbb) : (12ab - 5bb) =$ what?

23) $(72xxxx - 78xxxy - 10xxyy + 17xyyy + 3yyyy) : (6xx - 4xy - yy) =$ what?

24) $$\left\{\begin{matrix} 18aa + 33ab + 42ac - 12ad - 30bb + 124bc \\ + 8bd - 16cc - 32cd \end{matrix}\right\} : (6a + 15b - 2c - 4d) = \text{what?}$$

25) $$\left\{\begin{matrix} 108aa - 33ac - 9ad - 9ae - 24ab + 10bc \\ + 2bd + 2be - 5cc - cd - ce \end{matrix}\right\} : (12a - 5c - d - e) = \text{what?}$$

26) $$\left\{\begin{matrix} - 75aabfxx + 65aacxy + 60abxyy - 65axz \\ + 180abfxxy - 156acxyy - 144bxyyy + 156xyz \end{matrix}\right\} : (15abfx - 12byy - 13acy + 13z) = \text{what?}$$

27) $(aa - bb) : (a - b) =$ what?

28) $(4aa - 9bb) : (2a - 3b) =$ what?

29) $(16aa - 9bb) : (4a + 3b) =$ what?

30) $(a\,a\,a\,a - 9\,a\,a\,b\,b - 6\,a\,b\,c\,c - c\,c\,c\,c) : (a\,a - 3\,a\,b - c\,c) =$ what?

31) $(a\,a\,a\,a - b\,b\,b\,b) : (a - b) =$ what?

32) $(16\,a\,a\,a\,a - 81\,b\,b\,b\,b) : (2\,a - 3\,b) =$ what?

33) $(32\,a\,a\,a\,a\,a + b\,b\,b\,b\,b) : (2\,a + b) =$ what?

34) $(9\,a\,a\,b\,b - 25\,f\,f\,m\,m - 70\,d\,f\,m - 49\,d\,d) : (3\,a\,b + 5\,f\,m + 7\,d) =$ what?

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c. PARTIAL DIVISION, PERFORMED IN CASES WHERE THE DIVISOR IS NOT AN EXACT NUMBER OF TIMES CONTAINED IN THE DIVIDEND.

1) Find the First Term and Remainder of the Division $1 : (1 - b)$?

2) Find the Two First Terms and Remainder of the Division $1 : (1 - b)$?

3) Find the Three First Terms and Remainder of the Division $1 : (1 - b)$?

4) Find the Five First Terms of the Division $1 : (1 - b)$?

5) What is the Series obtained by the Division 1, by $1 - b$?

6) Find the First Term and Remainder of the Division $1 : (1 + b)$?

7) Find the Two First Terms and Remainder of the Division $1 : (1 + b)$?

8) Find the Four First Terms and Remainder of the Division $1 : (1 + b)$?

9) What is the Series obtained by the Division $1 : (1 + b)$?

10) Find the First Term and Remainder of the Division $a : (1 - a)$?

11) Find the Two First Terms and Remainder of the Division $a : (1 - a)$?

12) Find the Five First Terms and Remainder of the Division $a : (1 - a)$?

13) What is the Series obtained by Dividing a by $1 - a$?

14) Find the First Term and Remainder of the Division $a : (1 + a)$?

15) Find the Three First Terms and Remainder of the Division $a : (1 + a)$?

16) What is the Series obtained by Dividing a by $1 + a$?

17) Find the First Term and Remainder of the Division $a : (a - 1)$?

18) Find the Two First Terms and Remainder of the Division $a : (a - 1)$?

19) What is the Series obtained by Dividing a by $a - 1$?

20) Find the First Term and Remainder of the Division $a : (a + 1)$?

21) Find the Three First Terms and Remainder of the Division $a : (a + 1)$?

22) What is the Series obtained by Dividing a, by $a + 1$?

23) Find the First Term and Remainder of the Division $c : (c - b)$?

24) Find the Two First Terms and Remainder of the Division $c : (c - b)$?

25) Find the Three First Terms and Remainder of the Division $c : (c - b)$?

26) What is the Series obtained by Dividing c by $(c - b)$?

27) Find the First Term and Remainder of the Division $c : (c + b)$?

28) Find the Three First Terms and Remainder of the Division $c : (c + b)$?

29) What is the Series obtained by Dividing c by $c + b$?

30) If the Division c, by $(a - b)$ gives $\frac{c}{a}$, and leaves $+\frac{b\,c}{a}$ for a Remainder;

or also $\frac{c}{a} + \frac{b\,c}{a\,a}$ and leaves $\frac{b\,b\,c}{a\,a}$ for a Remainder,

or also $\frac{c}{a} + \frac{b\,c}{a\,a} + \frac{b\,b\,c}{a\,a\,a}$ and leaves $\frac{b\,b\,c}{a\,a\,a}$ for a Remainder, what is the Series obtained by Dividing c by $a - b$?

31) What is the First Term and Remainder of the Division $c : (a + b)$?

32) What are the Two First Terms and Remainder of the Division $c : (a + b)$?

33) What is the Series obtained by Dividing c by $a + b$?

34) What is the First Term and Remainder of the Division $1 + x$, by $1 - x$?

35) What are the Three First Terms and Remainder of the Division $1 + x$, by $1 - x$?

36) What is the Series obtained by Dividing $1 + x$, by $1 - x$?

SECTION II.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF POWERS.

1) a Multiplied by a gives what power of a?

2) $x \times x =$ what power of x?

3) a Multiplied twice by itself gives what power of a?

4) $x \times x \times x =$ what power of x?

5) a Multiplied Three times by itself gives what power of a?

6) $x \times x \times x \times x =$ what power of x?

7) If b be Multiplied 10 times by itself, what power of b is obtained?

8) Of how many factors a, does the 10th power of a consist?

9) Of how many factors b, does the 15th power of b consist?

10) How many factors x are contained in x^{14}?

11) How many factors x^2 are contained in x^{14}?

12) How many factors y^3 are contained in y^9?

13) What is a with the exponent o equal to?

14) What is a with the negative exponent 2, equal to?

15) What is a^{-3} equal to?

16) How may a with the negative exponent 2, be expressed by a fraction?

17) How may a^{-1} be expressed by a fraction ?
18) To what fraction is 2^{-1} equal ?
19) To what fraction is 3^{-1} equal ?
20) To what fraction is 2^{-2} equal ?
21) To what fraction is 2^{-2} equal ?
22) To what fraction is 4^{-1} equal ?
23) What is 2^0 equal to ?

A. *Addition of Powers.*

6. ADDITION OF SIMPLE QUANTITIES.

1) Add together $a^2 + 2\ a^2 + 3\ a^2$.
2) Add together $3\ a^2 + 5\ a^2 + 7\ a^2$.
3) Add together $4\ x^2 - 3\ x^2 + 5\ b^3 - 2\ b^3$.
4) Add together $7\ x^2 - 4\ a^2 + 3\ x^2 - 7\ a^2$.
5) Add together $5\ x^2 - 3\ y^2 + 3\ y^2 - 4\ x^2$.
6) Add together $4\ x^2 - 4\ y^2 + 2\ x^2 - 5\ y^2$.
7) Add together $3\ x^2 + 4\ y^2 - 5\ x^2 - 7\ y^2$.
8) Add together $2\ x^2 + 3\ x - 7\ x^2 - 15\ x$.
9) Add together $14\ x^2 + 13\ x^2\ y - 3\ x^2 + 4\ x^2\ y$.

10) $9\ x^2\ y + 10\ x\ y^2 + 3\ x^2\ y - 11\ x\ y^2 =$ what ?
11) $6\ x^2 - x^2\ y + 7\ x - 3\ x^2\ y + x =$ what ?
12) $10\ x^4 + 3\ x^3\ y + 9\ x^4 + x^4 - x^3\ y =$ what ?
13) $9\ x^2 - 10\ x^4 + y^5 - 8\ x^2 - x^2 + 10\ x^4 - y^5$
$=$ what ?
14) $10\ a^4 + 3\ a^4 + 6\ a^4 - a^4 - 5\ a^4 =$ what ?

15) $3.\, a^{-1} + 4\, a^{-1} =$ what?

16) $4\, a^{-2} + 3\, a^{-2} - 6\, a^{-2} =$ what?

17) $6^2 + 3.\, 5^2 + 2.\, 6^2 + 5^2 =$ what?

18) $6^4 + 2.\, 8^3 + 196^4 + 5.\, 8^3 =$ what?

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b. ADDITION OF COMPOUND QUANTITIES.

1) Add
$$\left\{\begin{array}{rrr} 5\,a^2 b & +\,3\,a\,b^2 & +\,c \\ -\,4\,a^2 b & +\,2\,a\,b^2 & -\,c \\ 3\,a^2 b & +\,3\,a\,b^2 & -\,c \end{array}\right.$$

2) Add
$$\left\{\begin{array}{rrr} 4\,a^2 b\,c & -\,2\,a^2 b\,d & -\,d \\ 3\,a^2 b\,c & +\,a^2 b\,d & -\,2\,d \\ a^2 b\,c & -\,a^2 b\,d & -\,3\,d \end{array}\right.$$

3) Add
$$\left\{\begin{array}{rrr} 5\,a^4 b & +\,3\,a^2 b^2 c & -\,7\,a\,b \\ -\,6\,a^4 b & +\,2\,a^2 b^2 c & +\,17\,a\,b \\ 9\,a^4 b & -\,8\,a^2 b^2 c & -\,10\,a\,b \end{array}\right.$$

4) Add
$$\left\{\begin{array}{rrr} 3\,a^4 b & -\,5\,a\,b^4 & +\,a^4 \\ 2\,a^4 b & -\,3\,a\,b^4 & +\,2\,a^4 \\ -\,5\,a^4 b & +\,8\,a\,b^4 & -\,3\,a^4 \end{array}\right.$$

5) Add
$$\left\{\begin{array}{rrr} 9\,a^3 b^2 c^4 & -\,7\,b & \\ 18\,a^3 b^2 c^4 & -\,3\,b & +\,3.\,2^4 \\ a^3 b^2 c^4 & +\,2\,b & -\,2.\,2^4 \end{array}\right.$$

6) Add
$$\left\{\begin{array}{lrrr} 7\,a^3 b\,c^2 & & & \\ 3\,a^3 b\,c^2 & -\,3\,a^2 b & +\,d & \\ & +\,2\,a^2 b & -\,2\,d & +\,5\,a\,b^2 \end{array}\right.$$

7) Add
$$\left\{\begin{array}{lrrr} 3\,a\,b\,x^2 & & & \\ 2\,a\,b\,x^2 & -\,3\,a & & \\ 3\,a\,b\,x^2 & -\,4\,a & -\,d & \\ 3\,a\,b\,x^2 & +\,2\,a & +\,d & +\,m \end{array}\right.$$

8) Add
$$\left\{\begin{array}{lrrr} 2\,a\,b^3 c & +\,4\,a\,b^2 c^2 & -\,3\,a^4 d^5 & \\ & -\,2\,a\,b^2 c^2 & +\,a^4 d^5 & \\ & & -\,2\,a^4 d^5 & -\,4\,a^2 \end{array}\right.$$

9) Add

$$\left\{\begin{array}{rrr} 4\,a^3\,b^2\,d & -\quad a^2\,b^3\,d & -\quad d^{-2}\\ 7\,a^3\,b^2\,d & -3\,a^2\,b^3\,d & -4\,d^{-2}\\ & +5\,a^2\,b^3\,d & -7\,d^{-2}\\ & & -3\,d^{-2}\end{array}\right.$$

10) Add

$$\left\{\begin{array}{rrrr} 45\,a^2\,b^3 & -\ 75\,a^2\,b^3\,c^{-1} & -\quad p^2 & +\quad d\\ & +25\,a^2\,b^3\,c^{-1} & +2\,p^2 & +2\,d\\ & & +3\,p^2 & +3\,d\\ & & & +4\,d\end{array}\right.$$

B. Subtraction of Powers.

a. SUBTRACTION OF SIMPLE QUANTITIES.

1) $\begin{array}{ll}\text{From} & a^2\\ \text{Subtract} & \underline{a^2}\end{array}$

2) $\begin{array}{ll}\text{From} & 3\,a^2\\ \text{Subtract} & \underline{2\,a^2}\end{array}$

3) $\begin{array}{ll}\text{From} & 10\,a^3\\ \text{Subtract} & \underline{10\,a^3}\end{array}$

4) $\begin{array}{ll}\text{From} & 7\,m^4\\ \text{Subtract} & \underline{11\,m^4}\end{array}$

5) $\begin{array}{ll}\text{From} & 6\,a^5\\ \text{Subtract} & \underline{4\,b^5}\end{array}$

6) $\begin{array}{ll}\text{From} & a^4\\ \text{Subtract} & \underline{-a^4}\end{array}$

7) $\begin{array}{ll}\text{From} & 9\,a^3\\ \text{Subtract} & \underline{-\,a^3}\end{array}$

8) $\begin{array}{ll}\text{From} & a^2\\ \text{Subtract} & \underline{-\,b^2}\end{array}$

9) $\begin{array}{ll}\text{From} & -\,10\,a^2\\ \text{Subtract} & \underline{-\,10\,a^2}\end{array}$

10) $\begin{array}{ll}\text{From} & -\,10\,a^3\\ \text{Subtract} & \underline{-\,11\,a^3}\end{array}$

11) $\begin{array}{ll}\text{From} & -\,6\,a^2\\ \text{Subtract} & \underline{-\,5\,a^2}\end{array}$

12) $\begin{array}{ll}\text{From} & -\,3\,b^2\\ \text{Subtract} & \underline{-\,5\,c^2}\end{array}$

13) $\begin{array}{ll}\text{From} & a^2\\ \text{Subtract} & \underline{-\,3\,a^2}\end{array}$

14) $\begin{array}{ll}\text{From} & -\,a^2\\ \text{Subtract} & \underline{-\,b^2}\end{array}$

15) $\begin{array}{ll}\text{From} & -\,c^2\\ \text{Subtract} & \underline{d^2}\end{array}$

16) $\begin{array}{ll}\text{From} & 3\,a^2\\ \text{Subtract} & \underline{a}\end{array}$

b. SUBTRACTION OF COMPOUND QUANTITIES.

1) From $4\,a^2 - 5\,b^2 + c^2$
Subtract $\underline{3\,a^2 - 4\,b^2 + c^2}$

2) From $14\,a^3 - 4\,b^2 + 17\,c^2 - d$
Subtract $\underline{3\,a^3 + 2\,b^2 + 15\,c^2 - d}$

3) From $17\,a^3 + 4\,b^2 + e^2$
Subtract $\underline{13\,a^3 - 7\,b^2}$

4) From $4\,a^3 - 12\,a\,b^2 - 3\,c\,d^2\,e^3$
Subtract $\underline{-3\,a^2 - 9\,a\,b^2 + 2\,c\,d^2\,e^3 - d\,f^2}$

5) From $12\,a^2 - 13\,b^2\,d^2 + c^3\,d$
Subtract $\underline{9\,a^2 + 3\,c^3\,d + k^2}$

6) From $14\,a^3 f + 9\,a^2\,b^2\,c^2$
Subtract $\underline{14\,a^3 f - a^2\,b^2\,c^2 - d^3 f + 4\,a^2\,b^2}$

7) From $9\,a^4\,b^5 - 9\,a^2\,b^5 + 3$
Subtract $\underline{-9\,a^4\,b^5 - 9\,a^2\,b^5 - 3\,a + d}$

8) From $a^2 + a^2\,b^2\,c^2$
Subtract $\underline{-a^2 - a^2\,b^2\,c^2 - 3\,a\,b^3}$

9) From $10\,a^3\,b^4 - 10\,a^2\,b^5\,c - 4\,d^4$
Subtract $\underline{9\,a^3\,b^4 + a^2\,b^5\,c - 3\,d^4}$

10) From $5\,a^4 - 7\,a^3\,b^2 - 3\,c\,d^2 + 7\,d$
Subtract $\underline{3\,a^4 - 15\,a^3\,b^2 - 7\,c\,d^2 - 3\,a^2}$

11) From $5\,a^{-4} - 7\,a^{-3}\,b^2 - 3\,c^{-1}\,d^2$
Subtract $\underline{3\,a^{-4} - 15\,a^{-3}\,b^2 - 7\,c^{-1}\,d^2}$

12) From $a^{-3} + 2\,a^{-4}\,d$
Subtract $\underline{-a^{-3} + a^{-4}\,d - d^3\,e + e\,f}$

C. Multiplication of Powers. *

a. MULTIPLICATION OF SIMPLE QUANTITIES.

1) Multiply a by a. 2) Multiply a by a^2.
3) Multiply a^2 by a^2. 4) Multiply a by a^3.
5) Multiply a^2 by a^3. 6) Multiply a^3 by a^3.
7) Multiply a^4 by a^2. 8) Multiply a^5 by a^3.
9) Multiply a^5 by a^5. 10) Multiply a^6 by a^7.
11) Multiply $3a^2 \times a^2$. 12) Multiply $4a^2 \times 2a^3$.
13) Multiply $5a^3 \times 4a^5$. 14) Multiply $5a^2 \times -a$.
15) Multiply $5a^3 \times -a^4$.
16) Multiply $-4a^5$ by $-3a^2$.
17) Multiply $-3a^3$ by $-a^2$.
18) Multiply $-4a^2$ by $5a^7$.
19) Multiply $-a^3$ by a^3.
20) Multiply $-a^3$ by $-a^3$.
21) Multiply $4a^2$ by a^3, and by a^2.
22) Multiply $3a^2$ by $2a^3$, and by a^3.
23) Multiply $3a^2$ by $-2a^2$, and by $-a^3$.
24) Multiply $-a^2$ by $-3a^3$, and by $-a^4$.

25) $5a^3 \times 4a^2b =$ what?
26) $5a^2 \times a^7 \times 3a^2 =$ what?
27) $5a^3 \times a^7 \times 7a^5 \times 3a^6 =$ what?
28) $5a^3 \times -a^7 \times -7a^5 \times -3a^6 =$ what?
29) $3a^2 \times 2a^7 \times 2a^5 \times -4a^5 =$ what?
30) $4a^5b \times 3a^3 \times -2a^5 \times -2a^4 =$ what?
31) $-4ab \times -3ab \times -a \times -b =$ what?

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* RULE. — Multiply the Coefficients and add the Exponent.

32) $2\ a^{-3} \times 7\ a^{-9} =$ what?*
33) $2\ a^{-3} \times 7\ a^{-9} \times 3\ a^{6} =$ what?
34) $2\ a^{-3} \times 7\ a^{-9} \times -3\ a^{6} =$ what?
35) $-2\ a^{-3} \times 7\ a^{-9} \times -3\ a^{6} =$ what?
36) $-3\ a^{-2} \times -4\ a^{-3} \times -4\ a^{5} =$ what?
37) $3.\ 2^{2} \times 4.\ 2^{3} =$ what?
38) $2.\ 3^{2} \times 3.\ 3^{2} =$ what?
39) $3.\ 4^{2} \times 2.\ 4^{2} \times 3.\ 4^{2} =$ what?
40) $3.\ 4^{-2} \times 2.\ 4^{-2} =$ what?
41) $4.\ 4^{-2} \times 5.\ 4^{-2} \times 4^{-2} =$ what?
42) $4.\ 4^{-2} \times -5.\ 4^{-2} \times 3.\ 4^{3} =$ what?
43) $3\ a^{2}\ b + 4\ a^{2}\ b =$ what?
44) $5\ a^{3}\ b^{4} \times 10\ a^{2}\ b^{5}\ c =$ what?
45) $5\ a^{3}\ b^{4} \times 10\ a^{2}\ b^{5}\ c \times -3\ a^{7} =$ what?
46) $3\ a^{3}\ b^{-4} \times 3\ a^{2}\ b^{-5}\ c \times -3\ a^{7} =$ what?
47) $-7\ a\ b^{4}\ c^{5} \times -3\ a^{2}\ b^{5}\ c =$ what?
48) $-7\ a\ b^{-4}\ c^{5} \times -3\ a^{2}\ b^{-5}\ c =$ what?
49) $-7\ a\ b^{-4}\ c^{-5} + -3\ a^{2}\ b^{-5}\ c^{-1} =$ what?
50) $5\ a^{3}\ b^{4} \times a^{2}\ b^{8} =$ what?
51) $5\ a^{3}\ b^{4} \times 3\ a^{2}\ b^{8} \times 4\ a\ c\ b =$ what?
52) $5\ a^{2}\ b^{4} \times 3\ a^{3}\ b^{5} + 4\ a\ b^{-3} =$ what?
53) $h^{4}\ l^{11}\ x \times h^{7}\ l^{9} =$ what?
54) $h^{4}\ l^{11}\ x \times h^{-7}\ l^{-9} =$ what?
55) $h^{4}\ l^{11}\ x \times h^{-7}\ l^{-9} =$ what?
56) $h^{4}\ l^{11}\ x \times h^{-7}\ l^{-9} \times 3\ h^{-2}\ l^{-6}\ x =$ what?
57) $-13\ a\ c^{3} \times -4\ a^{3}\ b^{6}\ c^{2} =$ what?
58) $-13\ a\ c^{3} \times -4\ a^{-3}\ b^{-6}\ c^{-2} =$ what?
59) $-13\ a^{-1}\ c^{-3} \times -4\ a^{-3}\ b^{-6}\ c^{-2} =$ what?
60) $a^{2}\ b^{2}\ c^{2} \times -3\ a^{2}\ b^{3}\ c^{4} =$ what?

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* The *negative* exponents are added like the positive ones.

61) $5\,a^4\,b^3\,c \times 4\,a^{-4}\,b^{-3}\,c^{-1} =$ what ? *
62) $13\,a^3\,b^4\,c \times -3\,a^{-3}\,b^4\,c^{-1} =$ what ?
63) $14\,a^2\,b^4\,c \times 3\,a^{-1}\,b^2\,c^2 \times 2\,a^{-2}\,b^{-6}\,c^{-3} =$ what ?
64) $3\,a^3\,b + c\,b^5 \times 3\,c\,b^{-5} =$ what ?
65) $4\,a^2\,b^5 \times 3\,c\,b^5 \times b^{-4} =$ what ?
66) $2\,(a + x)^2 \times 3\,(a + x)^3 =$ what ?

b. MULTIPLICATION OF COMPOUND QUANTITIES.

1) Multiply $a^2 + 3\,a$
by $2\,a$

2) Multiply $a^2 + 2\,a^3$
by $2\,a^2$

3) Multiply $a^2 + 2\,a + b^2$
by $3\,a$

4) Multiply $2\,a^2 - 3\,a^4 - c$
by $3\,a^2$

5) Multiply $a^2 - 3\,a + b^2$
by $4\,a$

6) Multiply $a^2 - 3\,a + 2\,b^2$
by $-4\,a^2$

7) Multiply $a^2 - 3\,a\,b - 5\,b^2$
by $4\,a^2\,b$

8) Multiply $2\,a^3\,b^5 - 4\,a^2\,b^3$
by $3\,a^2\,b^2\,c$

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* The pupil ought to recollect that $a^0 = 1$.

9) Multiply $2a^3b^5 - 5a^2c^6 + 9a^3b^2c^3$
by $3a^2bc^2$

10) Multiply $7a^5l + 2l^4 - 3ah^3 - 4$
by $8h^4l$

11) Multiply $7a^{-5}l + 2l^4 - 3a^{-1}h - 3$
by $-8h^4l^{-1}$

12) Multiply $a^3b^4 - cb^5d^3f + 3c^3$
by $2bc^2d$

13) Multiply $a^3b^4 - cb^5d^3f + 3c^3$
by $2b^{-2}d$

14) Multiply $a + b$
by $a + b$

15) Multiply $a - b$
by $a - b$

16) Multiply $a + b$
by $a - b$

17) Multiply $a^2 + 3b^2$
by $a - b$

18) Multiply $a^4 - 2b^3$
by $a - b$

19) Multiply $a^4 - 3b^3$
by $a^2 - b^2$

20) Multiply $x^2 - 3x - 7$
by $x - 2$

21) Multiply $3k^2 - 5kl + 2l^2$
by $k^2 - 7kl$

22) Multiply $6f^2 - 17fl + 3l^2$
by $f^3 + 4f^2l$

23) Multiply $4a^2 - 16ax + 3x^2$
by $5a^3 - 2a^2x$

24) Multiply $a^2 + a^4 + a^6$
by $a^2 - 1$

25) Multiply $a^4 - 2 a^3 b + 4 a^2 b^2 - 8 a b^3 + 16 b^4$, by $a + 2 b$.

26) Multiply $2 a^4 x^2 - 3 b^4 y^2$, by $2 a^2 x - 3 b^2 y$.

27) Multiply $2 a^4 x^2 - 3 b^4 y^2$, by $2 a^5 x^2 - 3 b^4 y^2$.

28) Multiply $7 a^3 - 5 a^2 b + 6 a b^2 - 2 b^3$, by $3 a^4 - 4 a^3 b + 16 a^2 b^2$.

29) Multiply $5 x^2 + 3 a x - 7 a^2$, by $2 x^2 - a x - a^2$.

30) Multiply $a^6 - 3 a^4 b^2 + 5 a^2 b^4$, by $7 a^4 - 4 a^2 b^2 + b^4$.

31) $(a^5 - 5 a^4 b + 10 a^3 b^2 + 10 a^2 b^3 - 5 a b^4 - b^5) \times (a^3 - 3 a^2 b + 3 a b^2 - b^3) =$ what?

32) $(a^2 + a z + z^2) \times (a^2 - a z + z^2) =$ what?

33) $(15 a^6 b^2 - 7 a^5 b^4 + 6 a^{-4} b^6) \times (8 a^2 b^2 - 3 a b^4) =$ what?

34) $(15 a^{-6} b^2 - 7 a^{-5} b^4 + 6 a^{-4} b^6) \times (8 a^{-2} b^2 - 3 a^{-1} b\ 4) =$ what?

35) $(3 x^2 y^7 - 2 x^2 y^5 + 8 x^6 y^3) \times (2 x^3 y^5 + 6 x y^3 + 12 x^5 y) =$ what?

36) $(13 a^5 b + 10 a^2 b^2 - 4 a b^3) \times (6 a^2 b^2 - 18 b^3 - 7 a^3 b^4) =$ what?

37) $(13 a^{-5} b + 10 a^{-2} b^2 - 4 a b^3) \times (6 a^{-3} b^2 - 18 b^3 - 5 a^3 b^4) =$ what?

38) $(5 a^3 b^3 c^2 - 6 a^4 b^2 c^5 + 7 a^8 b^5 c^6) \times (2 a^3 b^3 c^2 + 3 a^4 b^2 c^5 - 6 a^7 b^4 c^3) =$ what?

39) $(14 a^5 c^2 - 6 a^2 b c^2 + c^3) \times (14 a^5 c^2 + 6 a^2 b c^2 - c^3) =$ what?

40) $(a^2 + 2 c^3 d^4 - 7 c^2) \times (a^2 - 2 c^3 d^4 + 7 c^2) =$ what?

D. Division of Compound.[*]

a. DIVISION OF SIMPLE QUANTITIES.

1) Divide a by a 2) Divide a^2 by a^2
3) Divide a^5 by a^3 4) Divide a^6 by a^4
5) Divide a^7 by a^6 6) Divide $a^2 b$ by b
7) Divide $a^3 b^2 c$ by $a^2 b$ 8) Divide $a^5 b^4 c^3$ by $a^2 b^3 c$
9) Divide $-4 a^2$ by $2 a$ 10) Divide $-14 a^3$ by $7 a^2$.
11) Divide $-15 a^4 b$ by $-3 a^2$
12) Divide $-24 a^4 b^2$ by $6 a^3 b$
13) Divide $-36 a^4 b^3$ by $9 a^3$
14) Divide $-27 a^3 b^2$ by $-9 a b^2$
15) Divide $4 a^5 b^3 c$ by $-2 a^3 c$
16) Divide $3 a^5 b^2 c d$ by $-4 a^3 b c$

17) $8 a^{10} : 2 a^4 =$ what ?
18) $7 a^3 : 2 a =$ what ?
19) $14 a^{-8} : 2 a^{-2} =$ what ?
20) $3 a^{-8} : 3 a^{-1} =$ what ?
21) $9 a^{-7} : a =$ what ? [†]
22) $15 a^{-7} : 3 a =$ what ?
23) $c a^{18} : d a^6 =$ what ?
24) $c a^{18} : d a^{-6} =$ what ?
25) $16 a^6 b^4 : 4 a^{-3} b^{-2} =$ what ?
26) $16 a^{-6} b^{-4} : 4 a^{-7} b^{-5} =$ what ?
27) $14 a^3 b^2 c f g : 2 a^2 b^3 c k =$ what ?

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[*] Rule. — Divide the Coefficients and subtract the Exponent.

[†] The negative Exponent of the divisor, becomes by subtraction positive.

28) $15\ a^2\ b^3\ c : 16\ a^3 b^{-4}\ c =$ what ?

29) $-\ 25\ a\ b^3\ c^2 : -\ 15\ a^2\ b^{-1}\ c =$ what ?

30) $24\ a\ b\ c : -\ 16\ a^{-2}\ b^{-1}\ c =$ what ?

31) $36\ a^{-4}\ b\ c^3 : 6\ a^{-4}\ b\ c =$ what ?

32) $a^2\ b^3\ c : 4\ a\ b^4\ c =$ what ?

---

b. DIVISION OF COMPOUND QUANTITIES.

1) Divide $6\ a^3\ b^2 - 10\ a^2 f$, by $2\ a^2$

2) Divide $8\ a^5\ b^2 - 8\ a^2\ f^2 + 7\ a^5\ b\ x$, by $2\ a^2$

3) Divide $4\ a^2\ x^5 - 14\ a\ x^3 + 3\ a^3\ b^2\ x$, by $2\ a^3$

4) Divide $2\ a\ x^4 - 7\ a\ x^2 + 3\ a\ b^2\ x^2$, by $2\ a^3\ x^3$

5) Divide $4\ a^3\ x^5 + 2\ a^3\ x^2 - 3\ a\ b^2\ x$, by $-\ 2\ a^2\ x$

6) Divide $-\ 7\ x^3 + 14\ x^3\ y^3 - y^3$, by $x^3\ y^3$

7) Divide $15\ x^2 - 20\ x^2\ y^2 + y^2$, by $5\ x^2\ y^2$

8) Divide $9\ x\ y\ z + 27\ x^2\ y^2\ z^2 + x^3\ y^3\ z^3$, by $x^2\ y^2\ z^2$

9) Divide $32\ x^3\ y^3\ z^3 + 4\ x^6\ y^6\ z^6$, by $8\ x^4\ y^4\ z^4$

10) Divide $3\ a^2\ b^6 - 5\ a\ c + 2\ a^3\ c^2 - 2\ a^2\ c^2$, by $-2\ a^3\ b^2$

11) Divide $b\ c^3 - c^3\ x$, by $b - x$

12) Divide $a^2 + 2\ a\ b + b^2$, by $a + b$

13) Divide $a^3 + 3\ a^2\ b + 3\ a\ b^2 + b^3$, by $a + b$

14) Divide $a^2 + a^2\ b - a\ b^2 - b^3$, by $a - b$

15) Divide $a^4 + 4\ a^3\ b + 6\ a^2\ b^2 + 4\ a\ b^3 + b^4$, by $a + b$

16) Divide $3\ a^5 + 16\ a^4\ b - 33\ a^3\ b^2 + 14\ a^2 b^3$, by $a^2 + 7 a\ b$

17) Divide $a^7 - 6\ a^6\ b^3 + 14\ a^5\ b^6 - 12\ a^4\ b^9$, by $a^3 - 2\ a^2\ b^3$

18) Divide $a^4 - 2\ a^2\ b^2 + b^4$, by $a^2 - b^2$

19) Divide $a^6 - 2\ a^3\ b^3 + b^6$, by $a^3 - b^3$

20) Divide $a^8 + 2\,a^4\,b^4 + b^8$, by $a^4 + b^4$

21) Divide $a^2\,b\,x^8 - a^3\,b\,x^7 + a^5\,x^7 - 8\,a^6\,x^6 + 7\,a^7\,x^5$, by $a^2\,x^2 - a^3\,x$.

22) Divide $-a^8\,b^4 + 15\,a^{11}\,b^5 - 48\,a^{14}\,b^6 - 20\,a^{17}\,b^7$, by $-a^6\,b + 10\,a^9\,b^2$

23) Divide $a^8 - 16\,z^8$, by $a^2 - 2\,z^2$

24) Divide $2\,a^4 - 13\,a^3\,b + 31\,a^2\,b^2 - 38\,a\,b^3 + 24\,b^4$, by $2\,a^2 - 3\,a\,b + 4\,b^2$

25) $(4\,c^4 - 9\,b^2\,c^2 + 6\,b^3\,c - b^4)$
$: (2\,c^2 - 3\,b\,c + b^2) =$ what ?

26) $(4\,c^4 - 9\,b^2\,c^2 + 6\,b^3\,c - b^4)$
$: (2\,c^2 + 3\,b\,c - b^2) =$ what ?

27) $(-1 + a^3\,x^3) : (-1 + a\,x) =$ what ?

28) $\left\{\begin{array}{l} 5\,a^5\,b^3\,c^5 - 22\,a^4\,b^3\,c^6 + 5\,a^3\,b^3\,c^7 + \\ 12\,a^2\,b^3\,c^8 - 7\,a^2\,b^2\,c^8 + 28\,a\,b^2\,c^9 \end{array}\right\}$
$: (a^2\,b\,c^2 - 4\,a\,b\,c^3) =$ what ?

29) $(-2\,a^{-8}\,x^5 + 17\,a^{-4}\,x^6 - 5\,x^7 - 24\,a^4\,x^8)$
$: (2\,a^{-3}\,x^3 - 3\,a\,x^4) =$ what ?

30) $(a^6 + 2\,a^3\,z^3 + z^6) : (a^2 - a\,z + z^2) =$ what ?

31) $(a^6 - 16\,a^3\,x^3 + 64\,x^6) : (a^2 - 4\,a\,x + 4\,x^2) =$ what ?

32) $(a^8 - b^8) : (a - b) =$ what ?

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c. PARTIAL DIVISION, PERFORMED IN CASES IN WHICH THE DIVISOR IS NOT AN EXACT NUMBER OF TIMES CONTAINED IN THE DIVIDEND. *

1) What Series is obtained by Dividing a by $1 + x$?

2) What Series is obtained by Dividing a, by $1 - x$?

3) What Series is obtained by Dividing a, by $a + 1$?

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* See the problems, page 12.

4) What Series is obtained by Dividing a, by $x - 1$?
5) What is the First Term and Remainder of a, by $b + d$?
6) What is the Series obtained by Dividing a, by $b - d$?

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d. POWERS OF POWERS.

1) What is the Second power of a^2 ?
2) What is the Third power of a^2 ?
3) What is the Fourth power of a^3 ?
4) What is the Second power of $2 a^2$?
5) What is the Third power of $5 a b^2$?
6) What is the Fourth power of $2 a^3 b^2$?
7) What is the Second power of $3 a^2 b^2 c^2$?
8) What is the Third power of a^3 ?
9) What is the Second power of $- a^2$?
10) What is the Third power of $- a^2$?
11) What is the Fourth power of $- a^2$?
12) What is the Fifth, the Sixth, the Seventh, the Eighth power of $- a^2$?
13) Raise $4 a^2 b^3 c$ to the Third power.
14) Raise $- 2 a^2 b^3 c^2$ to the Third power.
15) Raise $- 4 a^2 b^3 c^3$ to the Third power.
16) Raise $a^3 b^4 c^5$ to the Fifth power.
17) Raise $- 3 a^2 b$ to the Sixth power.
18) Raise $4 a^2 b^3 c^4 f g$ to the Square.

19) $(3 a^2)^2 =$ what ?
20) $(2 a^2 b^3)^3 =$ what ?
21) $(- 2 a^2)^2 =$ what ?
22) $(- 2 a^2)^3 =$ what ?

23) $\left((4\,a^2)^2\right)^3 =$ what?* 24) $(-\,a^2)^5 =$ what?

25) $(-\,b^{-3})^4 =$ what? 26) $\left((2\,a^3)^3\right)^4 =$ what?

27) $\left[\left((a^2)^3\right)^4\right]^5 =$ what? 28) $\left[\left((-a)^3\right)^4\right]^5 =$ what?

29) $(-\,a)^{-3} =$ what? 30) $(-\,a)^{-4} =$ what?

31) $(-\,a^2)^{-5} =$ what? 32) $(-\,a^3)^{-4} =$ what?

33) $\left((-\,a)^{-3}\right)^{-5} =$ what? 34) $\left((-\,2\,a)^2\right)^{-5} =$ what?

35) $\left((-2\,a^{-2})\right)^{-4} =$ what?

36) $\left((-2a)^{-3}\right)^{-5} =$ what?

SECTION III.

ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION, AND RAISING TO POWERS OF FRACTIONS.

A. Addition.

a. ADDITION OF SIMPLE QUANTITIES.

1) Add $\frac{a}{b}, \frac{2\,a}{b}, \frac{3\,a}{b}$

2) Add $\frac{b}{2\,c}, \frac{3\,b}{2\,c}, \frac{5\,b}{2\,c}$.

---

* Raise 4 a^2 first to the 2d power, and the answer again to the third. In the same manner are the following sums done.

3) Add $\frac{3a}{4}$, $\frac{5a}{4}$, $\frac{a}{4}$.

4) Add $\frac{4}{5}a^2, -\frac{3}{8}a^2, -\frac{1}{6}a^2 + \frac{2}{3}a^2$.

5) Add $\frac{3}{4}a^2 + \frac{1}{2}a^2 - 1\frac{4}{5}b^3 - 2\frac{1}{5}b^3$.

6) Add $\frac{3a^2}{b} + \frac{4a^2}{b^2} - \frac{5a^2}{b^2}$.

7) Add $\frac{5a^3}{b^4} - \frac{7a^3}{b^4} + \frac{11a^3}{b^4} - a^4$.

8) Add $\frac{5a^3}{x^3} - \frac{7a^3}{x^3} + a^4 + \frac{2a^4}{5}$.

9) Add $\frac{5}{4}a^4 - \frac{3}{2}a^4 - a^4 + 4a^4$.

10) Add $3ab^2x^3 + \frac{2ab^2x^3}{7} + \frac{3ab^2x^3}{8} =$ what ?

11) Reduce $\frac{a}{b} + \frac{c}{d}$ to a Common Denominator.

12) Reduce $\frac{a}{b} + \frac{2c}{2d}$ to a Common Denominator.

13) Reduce $\frac{a}{b} + \frac{b}{c} + \frac{c}{d}$ to a Common Denominator.

14) Reduce $\frac{a}{b} + \frac{b}{2c} + \frac{c}{3d}$ to a Common Denominator.

15) Reduce $\frac{a}{2b} + \frac{b}{4c} + \frac{c}{6d}$ to a Common Denominator.

16) Reduce $\frac{2a}{3b} + \frac{3b}{4c} + \frac{4c}{5d}$ to a Common Denominator.

17) Reduce $\frac{2a}{3bc}+\frac{3b}{4bc}+\frac{4c}{5bd}$ to a Common Denominator.

18) Reduce $\frac{1}{a}+\frac{1}{b}-\frac{1}{c}$ to a Common Denominator.

19) Reduce $\frac{1}{2a}+\frac{1}{3b}+\frac{1}{4c}$ to a Common Denominator.

20) Reduce $\frac{ab}{cd}-\frac{b}{de}+\frac{1}{f}$ to a Common Denominator.

21) Reduce $\frac{3a}{bc}+\frac{2b}{bcd}-\frac{2c}{3bcd}$ to a Common Denominator.

22) Reduce $\frac{a}{b}+\frac{b}{c}+\frac{d}{f}+\frac{g}{h}$ to a Common Denominator.

23) Reduce $\frac{a}{b}+\frac{c}{2d}+\frac{d}{3e}+\frac{f}{4g}$ to a Common Denominator.

24) Reduce $\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{f}+g+h$ to a Common Denominator.

---

b. ADDITION OF COMPOUND QUANTITIES.

1) Add $\left\{\begin{array}{rrrr} 3a & +\frac{1}{2}b & -3c & +\frac{1}{4}d \\ \frac{1}{2}a & -\frac{2}{3}b & +\frac{1}{5}c & -2d \\ & \frac{1}{6}b & +\frac{1}{10}c & -3d \end{array}\right.$

2) Add $\left\{\begin{array}{rr} \frac{3}{4}f & +\frac{3}{5}a \\ \frac{1}{2}f & -2a \\ f & -\frac{4}{7}a \\ & -\frac{1}{2}a \end{array}\right.$

3) Add
$$\left\{\begin{array}{l} 4\tfrac{1}{2}\,a - 3\tfrac{1}{3}\,b + c \\ 3\tfrac{2}{3}\,a + 2\tfrac{1}{2}\,c \\ \tfrac{1}{5}\,a + 2\tfrac{4}{5}\,b \\ + \tfrac{1}{2}\,b + 3\tfrac{1}{2}\,c - f \end{array}\right.$$

4) Add
$$\left\{\begin{array}{l} \tfrac{2}{3}\,a + \tfrac{4}{5}\,b + \tfrac{1}{2}\,d + 3\,g \\ \tfrac{1}{2}\,a - \tfrac{1}{3}\,b + \tfrac{3}{8}\,d - \tfrac{1}{2}\,g \\ + \tfrac{2}{3}\,b + \tfrac{1}{6}\,d - 1\tfrac{1}{2}\,g \\ - d - g \end{array}\right.$$

5) Add
$$\left\{\begin{array}{l} \frac{1}{4} + \frac{1}{b} + \frac{1}{bf} + \frac{1}{ab} \\ + \frac{1}{2b} + \frac{1}{2bf} + \frac{1}{2ab} \\ + \frac{3}{2bf} + \frac{1}{3ab} \end{array}\right.$$

6) Add
$$\left\{\begin{array}{l} \frac{a}{b} + \frac{c}{d} + \frac{e}{f} + \frac{g}{h} \\ + \frac{c}{2d} + \frac{e}{2f} + \frac{g}{2h} \\ + \frac{e}{3f} + \frac{g}{3h} \end{array}\right.$$

7) Add
$$\left\{\begin{array}{l} + \frac{1\,c^2}{2\,d^2} + \frac{1\,e^3}{3\,f^3} + \frac{1\,g^4}{4\,h^4} \\ + \frac{1\,e^3}{4\,f^3} + \frac{1\,g^4}{4\,h^4} \\ + \frac{1\,g^3}{4\,h^4} \end{array}\right.$$

8) Add
$$\left\{\begin{array}{l} \frac{2a}{3} + \frac{3bc}{4} + \frac{4de}{5} + \frac{fgh}{3} \\ + \frac{2bc}{5} + \frac{4de}{5} + \frac{2fgh}{5} \\ + \frac{3de}{7} - \frac{2fgh}{3} \\ - \frac{2fgh}{5} \end{array}\right.$$

9) Add $$\left\{\begin{array}{l} \dfrac{b^2 c^3}{7 g^2 h^3} + \dfrac{f^3 g}{8 m^3 n^3} \\ \dfrac{2 c^3 d}{7 g^2 h^3} - \dfrac{3 f^2 g}{8 m^3 n^3} \\ \dfrac{3 a^2 c^3}{2 g^2 h^3} - \dfrac{l^2 m}{8 m^3 n^3} \end{array}\right.$$

10) Add $$\left\{\begin{array}{l} \dfrac{a b}{c} - \dfrac{c d}{e} + \dfrac{e f}{g} \\ \dfrac{a b^2}{c^2} - \dfrac{c^2 d}{e^3} + \dfrac{e^4 f}{g^4} \\ \dfrac{a b^3}{c^3} + \dfrac{c^3 d}{e^5} - \dfrac{e^5 f^2}{g^6} \end{array}\right.$$

B. Subtraction.

b. SUBTRACTION OF SIMPLE QUANTITIES.

1) From $12\frac{1}{2}$
Subtract $3\frac{3}{4}$

2) From $-12\frac{1}{2}$
Subtract $3\frac{3}{4}$

3) From $-4\frac{1}{5}$
Subtract $-2\frac{2}{3}$

4) From $4\frac{1}{5}$
Subtract $-2\frac{1}{3}$

5) From $\frac{14}{15}$
Subtract $\frac{3}{5}$

6) From $-17\frac{11}{12}$
Subtract $-16\frac{17}{18}$

7) From $\frac{4}{5} a$
Subtract $\frac{2}{3} a$

8) From $-\frac{2}{3} b$
Subtract $-\frac{1}{4} b$

9) From $\frac{2}{3} b$
Subtract $-\frac{3}{4} b$

10) From $-\frac{4}{5} b$
Subtract $\frac{1}{4} b$

11) Reduce $\frac{a}{b} - \frac{c}{d}$ to a Common Denominator.

12) Reduce $\frac{a}{b} + \frac{c}{d} - \frac{e}{f}$ to a Common Denominator.

13) Reduce $\frac{a}{2b} - \frac{3c}{4d}$ to a Common Denominator.

14) Reduce $\frac{a}{2b} - \frac{3c}{4d} - \frac{4e}{5f}$ to a Common Denominator.

---

b. COMPOUND SUBTRACTION.

1) From $2a - 3\frac{1}{2}b + 4\frac{4}{5}d$
Subtract $\underline{-3\frac{1}{2}a - 2\frac{3}{8}b - 1\frac{5}{6}d}$

2) From $17\frac{1}{5}a - 31\frac{4}{7}b - 13\frac{4}{5}d$
Subtract $\underline{4\frac{3}{4}a - 9\frac{1}{3}b + 9\frac{2}{7}d}$

3) From $3\frac{1}{2}a + 4\frac{5}{8}d - \frac{7}{8}c - \frac{2}{3}g$
Subtract $\underline{2\frac{5}{6}a - 3\frac{1}{2}d - 4\frac{2}{3}c + \frac{7}{8}g}$

4) From $3\frac{1}{2}a - 5\frac{1}{4}l$
Subtract $\underline{-4\frac{3}{4}a + \frac{1}{2}\cdot m}$

5) From $\frac{1}{2}a^2 + \frac{3}{4}a^2b^2c$
Subtract $\underline{-\frac{3}{4}a^2 - \frac{1}{2}a^2b^2c^2 + \frac{3}{4}ab^3}$

6) From $\frac{1}{2}a^{-3} + 2a^{-4}d + \frac{d^3}{e} + \frac{e}{f}$

Subtract $\underline{-\frac{1}{4}a^{-3} + a^{-4}d - \frac{d^3}{e} + \frac{e}{3f}}$

7) From $\frac{1}{a} + \frac{1}{b}$

Subtract $\frac{1}{2a} + \frac{1}{2b}$

8) From $\frac{a}{b} + \frac{c}{d}$

Subtract $\frac{a}{2b} - \frac{c}{2d}$

9) From $\frac{ab}{cd} + \frac{ef}{gh}$

Subtract $-\frac{mn}{3cd} + \frac{ap}{4gh}$

10) From $\frac{ab}{2cd} - \frac{gh}{3mn}$

Subtract $\frac{a}{4cd} - \frac{h}{mn}$

11) From $a + b$

Subtract $\frac{1}{a} + \frac{1}{b}$

12) From $cd + fg + h$

Subtract $\frac{b}{cd} - \frac{e}{fg} + \frac{k}{h}$

C. Multiplication.

a. MULTIPLICATION OF SIMPLE QUANTITIES.

1) Multiply $\frac{1}{2}a$ by 3.
2) Multiply $\frac{2}{3}a$ by 4.
3) Multiply $17a$ by $\frac{3}{4}b$.
4) Multiply $\frac{3a}{2}$ by $\frac{5f}{4}$.
5) Multiply $\frac{3}{8}a$ by $-\frac{2b}{3}$.

6) Multiply $-\frac{5a}{4}$ by $-\frac{3b}{7}$.

7) Multiply $-\frac{abd}{2}$ by $\frac{cde}{4}$.

8) Multiply $\frac{cd}{4}$ by $\frac{3cfg}{8}$.

9) Multiply $23\,deg$ by $3\frac{3}{4}$.

10) Multiply $\frac{1}{4}ab$ by $\frac{2}{3}cd$.

11) Multiply $\frac{2}{3}ab$ by $\frac{4}{5}def$.

12) Multiply $\frac{4}{5}abc$ by $\frac{3}{7}def$.

13) Multiply $-\frac{3h}{5fg}$ by $\frac{5fg}{3h}$.

14) Multiply $-\frac{4hf}{5gm}$ by $-\frac{3hl}{4fm}$.

15) $\frac{1}{a} \times \frac{1}{b} =$ what?

16) $\frac{a}{b} \times \frac{c}{d} =$ what?

17) $\frac{1}{a} \times \frac{1}{b} \times \frac{1}{c} =$ what?

18) $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} =$ what?

19) $\frac{2a}{3b} \times \frac{4c}{5d} \times \frac{3e}{4f} =$ what?

20) $3\frac{1}{2}ab \times 2\frac{2}{5}d \times df =$ what?

21) $\frac{5ab}{2} \times \frac{2d}{5} \times \frac{7df}{8} =$ what?

22) $\frac{3ad}{bg} \times \frac{5bg}{2dl} \times \frac{2l}{5a} =$ what?

23) $\frac{1}{a} \times \frac{3ac}{x} =$ what?

24) $\frac{1}{a} \times \frac{3ac}{x} \times \frac{c}{x} =$ what?

25) $\frac{1}{3ab} \times 4cd \times \frac{1}{fg} =$ what?

26) $\frac{1}{4a^2b} \times 3c^2d^4 \times \frac{1}{f^2} =$ what?

27) $\frac{3a^2}{b^2} \times -\frac{2a^3}{b} \times -\frac{a}{b^2} =$ what?

28) $\frac{4a^4b^5}{5c^4d^3} \times \frac{3a^2b^4}{4c^4d^3} =$ what?

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b. COMPOUND MULTIPLICATION.

1) Multiply $\frac{3}{2}a - \frac{3}{4}bc - \frac{4}{5}df$
by $\underline{\hspace{8em}\frac{1}{3}b}$

2) Multiply $1\frac{1}{2}a - 4\frac{2}{3}de - 5\frac{4}{5}fg$
by $\underline{\hspace{9em}\frac{2}{3}ad}$

3) Multiply $\frac{5a}{2} + \frac{4b}{5} - \frac{c}{3}$
by $\underline{\hspace{8em}\frac{1}{2}}$

4) Multiply $\frac{b}{2} - \frac{c}{5} + \frac{d}{6}$
by $\underline{\hspace{8em}\frac{1}{3}}$

5) Multiply $\frac{3c}{4} - \frac{4d}{5} + \frac{5e}{6}$

by $\underline{\hspace{10em}\frac{2}{3}}$

6) Multiply $\frac{5a}{b} - \frac{13c}{2d} - \frac{6h}{5b}$

by $\underline{\hspace{10em}4b}$

7) Multiply $\frac{5a}{6b} - \frac{2c}{5d} + \frac{3h}{7g} + 6d$

by $\underline{\hspace{10em}\frac{3a}{4d}}$

8) Multiply $\frac{3a}{4b} - \frac{c}{4d} + \frac{2h}{5g} - \frac{4}{5}$

by $\underline{\hspace{10em}-\frac{1}{3a}}$

9) Multiply $\frac{2}{3}a + \frac{5}{2}b - 2\frac{1}{2}c$

by $\underline{\hspace{6em}\frac{1}{2}a - \frac{1}{3}b}$

10) Multiply $\frac{2}{3}a + \frac{4}{5}b$

by $\underline{\frac{1}{2}a - \frac{2}{3}b}$

11) $(\frac{4}{5}a^2 - \frac{3}{4}b^2) \times (\frac{1}{2}a^2 - \frac{1}{3}b^2) =$ what?

12) $(3a - 2\frac{1}{2}b) \times (2\frac{1}{5}a - 3\frac{3}{4}b) =$ what?

13) $(\frac{1}{2}a + \frac{2}{5}b - \frac{7}{2}c) \times (\frac{1}{2}a - b + \frac{1}{4}c) =$ what?

14) $(\frac{5}{2}x^2 + 3ax - \frac{7}{3}a^2) \times (2x^2 - ax - \frac{1}{2}a) =$ what?

15) $(\frac{1}{2}a + \frac{1}{3}b + \frac{1}{4}c) \times (\frac{1}{2}a + \frac{1}{3}b - \frac{1}{4}c) =$ what?

16) $(\frac{1}{2}a + \frac{1}{3}b + \frac{1}{4}c - \frac{1}{5}d) \times (\frac{1}{2}a + \frac{1}{3}b + \frac{1}{4}c - \frac{1}{5}d)$
$=$ what?

D. Division.

a. DIVISION OF SIMPLE QUANTITIES.

1) Divide $\frac{a}{b}$, by 2.

2) Divide $\frac{a}{2\,b}$ by 3.

3) Divide $\frac{a}{b}$, by $-c$.

4) Divide $\frac{a}{b}$, by $\frac{c}{d}$.

5) Divide d, by $\frac{e}{f}$.

6) Divide $f g$, by $\frac{h}{i}$.

7) Divide $3\,a\,b$, by $\frac{c}{2\,d}$.

8) Divide $\frac{1}{a}$ by $\frac{1}{b}$.

9) Divide $\frac{1}{2\,a}$ by $\frac{1}{2\,b}$

10) Divide $\frac{4\,a^2}{9\,b^3}$ by $-\frac{2\,a}{3\,b}$.*

11) Divide $\frac{4\,a^2\,b^2}{5\,c^3\,d^3}$ by $\frac{2\,a\,b}{c^2}$.

12) Divide $\frac{6\,f^2\,m^4}{g^4}$ by $\frac{3\,f\,m^2}{g^2}$

13) Divide $\frac{4}{m^2\,n^2}$ by $\frac{2}{m\,n}$.

14) Divide $3 f m$, by $-\frac{a\,m}{f\,g}$.

15) Divide $\frac{3}{4}\,a\,c$, by $\frac{5}{6}\,a\,b\,d$.

16) Divide $\frac{1}{7} f\,g\,g$, by $f\,g$.

17) Divide $\frac{4}{7}\,f^4\,g^6$, by $\frac{2}{7} f^2\,g^3$.

18) Divide $\frac{1}{4\,g^6}$ by $\frac{1}{2\,g^3}$.

19) Divide $\frac{15\,a^4\,b^8}{7\,c^3}$ by $5\,a^2\,b^4$

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* In such cases it is best to divide Numerator by Numerator, and Denominator by Denominater.

20) Divide 1, by $\frac{a}{b}$

21) $1 : \frac{2\,a}{3\,b} =$ what?

22) $-1 : \frac{4\,a^2\,b}{5\,c^3} =$ what?

23) $-3 : -\frac{2\,a\,b^3}{3\,d} =$ what?

24) $4 : -\frac{4\,a^3\,b}{5\,c\,d} =$ what?

25) $-4 : \frac{2\,a^2\,b}{3} =$ what?

26) $4\,a^2\,b^4\,c^3 : \frac{2\,a^2\,b^3}{c} =$ what?

E. Reduction of Fractions to the Lowest Term.

1) Reduce $\frac{4\,a}{6\,b}$

2) Reduce $\frac{4\,a^2\,c^2}{6\,c^3}$

3) Reduce $\frac{6\,a^4\,b^2}{8\,a^2\,b^4}$

4) Reduce $\frac{8\,a\,b}{4\,b\,c}$

5) Reduce $\frac{5\,x^3\,y}{15\,y^3}$

6) Reduce $\frac{14\,a^2\,y^2}{21\,a^2\,y^3}$

7) Reduce $\frac{27\,a^2}{81\,a}$

8) Reduce $\frac{124\,a^3\,b}{24\,a^2\,b^2}$

9) Reduce $\frac{4\,a^2\,b\,c}{6\,a\,b^2\,c^3}$

10) Reduce $\frac{18\,a\,b^2\,c^3}{21\,a^3\,b^2\,c}$

11) Reduce $\frac{4}{8\,a\,b}$

12) Reduce $\frac{16\,a^4}{32\,a^3\,b}$

13) Reduce $\frac{4\,a^4\,b^5}{5\,a^5\,b^5}$ 14) Reduce $\frac{3\,a^2\,b^4}{6\,a\,b^5\,c}$

15) Reduce $\frac{x\,y}{x^2\,y^2}$ 16) Reduce $\frac{x^2\,y^3}{x^3\,y^4}$

17) Reduce $\frac{14\,a^4\,y^3}{7\,a^2\,y}$ 18) Reduce $\frac{25\,a^3\,b^4}{5\,a^2\,b^3}$

19) Reduce $\frac{24\,a^4\,b\,c^5\,d}{6\,a^3\,b\,c^4}$ 20) Reduce $\frac{27\,a^4\,b^3\,c^2\,d^4}{18\,a^3\,b^2\,c\,d^3}$

F. Raising of Fractions to Powers.

1) Raise $\frac{a}{b}$ to the 2d power.

2) Raise $\frac{3\,a}{4\,b}$ to the 2d power.

3) Raise $\frac{3\,a^2}{4\,b^3}$ to the 3d power.

4) Raise $\frac{1}{a}$ to the 2d power.

5) Raise $-\frac{1}{a^2}$ to the 4th power.

6) Raise $\frac{1}{3\,a^2\,b^5}$ to the 2d power.

7) What is the 3d power of $\frac{1}{2\,a^2}$?

8) What is the 3d power of $-\frac{3}{4f^2}$?

9) What is the 4th power of $-\frac{2\,a^2\,b}{3}$?

10) What is the 3d power of $\frac{4}{3\,a^2}$?

11) $\left(-\frac{a}{b}\right)^2$ = what? 12) $\left[\left(-\frac{a}{b}\right)^3\right]^4$ = what?

13) $\left[\left(-\frac{a}{b}\right)^3\right]^{-4}$ = what?

14) $\left(\frac{a^2\,b^3}{c^4\,d^2}\right)^2$ = what? 15) $\left(\frac{d^4\,b^5}{c^3\,d\,f}\right)^4$ = what?

16) $\left(\frac{3\,a^3\,b^2}{4\,d^4}\right)^3$ = what? 17) $\left(-\frac{1}{3\,d^2\,e}\right)^2$ = what?

18) $\left(-\frac{3}{4\,a^2\,b^2}\right)^3$ = what? 19) $\left(-\frac{5}{6\,a^3\,b^3}\right)^{-4}$ = what?

20) $\left(-\frac{2\,a^2\,b^3}{3\,c^3\,d^4}\right)^2$ = what?

SECTION IV.

OF ROOTS.

A. Of Square Roots.

a. SQUARE ROOTS OF NUMBERS.

1) Extract the Square Root of 16.
2) Extract the Square Root of 100.
3) Extract the Square Root of 121.
4) Extract the Square Root of 256.
5) Extract the Square Root of 576.
6) Extract the Square Root of 4096.

7) Extract the Square Root of 9604.
8) Extract the Square Root of 61009.
9) Extract the Square Root of 582169.
10) Extract the Square Root of 254016.
11) Extract the Square Root of 37830.
12) Extract the Square Root of 902500.
13) Extract the Square Root of 57198969.
14) Extract the Square Root of 68492176.
15) Extract the Square Root of 25836889.
16) Extract the Square Root of 236144689.
17) Extract the Square Root of 1607448649.
18) Extract the Square Root of 7808111249.
19) Extract the Square Root of 14209130025.
20) Extract the Square Root of 41605800625.
21) Extract the Square Root of 1600400025.
22) Extract the Square Root of 48303584206084.
23) Extract the Square Root of 120888683790025.

24) $\sqrt{5}$ = what ? *
25) $\sqrt{13}$ = what ?
26) $\sqrt{22}$ = what ?
27) $\sqrt{96}$ = what ?
28) $\sqrt{153}$ = what ?
29) $\sqrt{101}$ = what ?
30) $\sqrt{7.65}$ = what ?
31) $\sqrt{9.6}$ = what ?
32) $\sqrt{15.2379}$ = what ?
33) $\sqrt{0.056}$ = what ?
34) $\sqrt{0.00789}$ = what ?
35) $\sqrt{0.003}$ = what ?
36) $\sqrt{0.014}$ = what ?
37) $\sqrt{\frac{1}{4}}$ = what ?
38) $\sqrt{\frac{9}{16}}$ = what ?
39) $\sqrt{\frac{64}{81}}$ = what ?
40) $\sqrt{\frac{256}{261}}$ = what ?
41) $\sqrt{\frac{121}{256}}$ = what ?
42) $\sqrt{\frac{256}{576}}$ = what ?
43) $\sqrt{\frac{15129}{54756}}$ = what ?
44) $\sqrt{\frac{210681}{322624}}$ = what ?
45) $\sqrt{\frac{1512900}{6451600}}$ = what ?

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* The extraction ought to be carried on to 5 decimals.

46) $\sqrt{\frac{378380304}{551075625}}$ = what? 47) $\sqrt{\frac{7}{4}}$ = what? *
48) $\sqrt{1\frac{4}{9}}$ = what? 49) $\sqrt{3\frac{1}{4}}$ = what?
50) $\sqrt{11\frac{11}{16}}$ = what? 51) $\sqrt{2\frac{14}{25}}$ = what?
52) $\sqrt{7\frac{13}{36}}$ = what? 53) $\sqrt{3\frac{15}{49}}$ = what?
54) $\sqrt{\frac{2}{3}}$ = what? † 55) $\sqrt{\frac{4}{5}}$ = what?
56) $\sqrt{\frac{5}{3}}$ = what? 57) $\sqrt{\frac{7}{8}}$ = what?
58) $\sqrt{\frac{5}{12}}$ = what? 59) $\sqrt{\frac{1}{17}}$ = what?
60) $\sqrt{\frac{3}{10}}$ = what?

b. EXTRACTION OF THE SQUARE ROOT OF ALGEBRAIC QUANTITIES.

1) Extract the Square Root of a^2.
2) Extract the Square Root of a^4.
3) Extract the Square Root of a^6.
4) Extract the Square Root of a^{10}.
5) Extract the Square Root of $4\,a^8$.
6) Extract the Square Root of $16\,a^2\,b^4$.
7) Extract the Square Root of $64\,a^4\,b^6$.
8) Extract the Square Root of $100\,a^6\,b^6$.
9) Extract the Square Root of $81\,a^2\,b^2\,c^2$.
10) Extract the Square Root of $121\,a^4\,b^2\,c^6$.
11) Extract the Square Root of $256\,a^4\,b^{16}$.
12) Extract the Square Root of $144\,x^2\,g^6\,z^8$.
13) Extract the Square Root of $9\,a^4\,b^2 f^{12}\,g^8$.
14) Extract the Square Root of $\frac{a^2}{b^2}$.

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* Extract the Root of Numerator and Denominator; or reduce the Vulgar Fraction first to a Decimal, and then extract the Root.

† Multiply Numerator and Denominator by 3; or reduce the Vulgar Fraction to a Decimal.

15) Extract the Square Root of $\frac{a^2\,b^2}{c^4}$.

16) Extract the Square Root of $\frac{4\,a^2}{b^2}$.

17) Extract the Square Root of $\frac{9\,a^2}{16\,b^4}$.

18) Extract the Square Root of $\frac{4\,a^2\,b^2}{9\,c^4\,d^8}$.

19) Extract the Square Root of $\frac{1}{a^2}$.

20) Extract the Square Root of $\frac{1}{4\,a^2}$.

21) Extract the Square Root of $\frac{1}{9\,a^2\,b^4}$.

22) $\sqrt{\frac{1}{9\,x^2}} =$ what ? 23) $\sqrt{\frac{4}{9\,a^2\,b^2}} =$ what?

24) $\sqrt{\frac{36\,a^6}{49\,b^2}} =$ what ? 25) $\sqrt{\tfrac{1}{9}\,a^6} =$ what ?

26) $\sqrt{\tfrac{1}{16}\,a^4\,b^2\,c^2} =$ what ?

27) Extract the Square Root of $a^2 + 2\,a\,b + b^2$.

28) $\sqrt{(a^2 - 2\,a\,b + b^2)} =$ what ?

29) Extract the Square Root of $a^2 - a\,b + \frac{b^2}{4}$.

30) $\sqrt{(x^2 + 2\,x + 1)} =$ what ?

31) Extract the Square Root of $x^2 - 2\,x + 1$.

32) $\sqrt{(f^6 + 6\,f^3\,x^4 + 9\,x^8)} =$ what ?

33) Extract the Square Root of $f^6 - 6 f^3\,x^4 + 9\,x^8$.

34) $\sqrt{\left(\frac{9\,a^8}{4} + 2\,a^4\,n^3 + \frac{4\,n^6}{9}\right)} =$ what?

35) $\sqrt{\left(\frac{9\,a^8}{4} - 2\,a^4\,n^3 + \frac{4\,n^6}{9}\right)} =$ what ?

36) $\sqrt{(\frac{25}{4}\,a^2\,b^2 - \frac{5}{3}\,a\,b\,c^2 + \frac{1}{9}\,c^4)} =$ what ?

37) $\sqrt{(x^4 - a\,x^3 + \frac{1}{4}\,a^2\,x^2)} =$ what ?

38) $\sqrt{(a^4 + 2\,a^2\,b^2 + b^4)} =$ what ?

39) $\sqrt{(a^8 + 2\,a^4\,b^4 + b^8)} =$ what ?

40) $\sqrt{(x^{10} + 2\,a^5\,y^5 + y^{10})} =$ what ?

41) $\sqrt{\left(\frac{a^2}{b^2} - \frac{4\,a}{3\,c} + \frac{4\,b^2}{9\,c^2}\right)} =$ what ?

42) $\sqrt{\left(\frac{a^2}{b^2} - \frac{2\,a\,c}{b\,d} + \frac{c^2}{d^2}\right)} =$ what ?

43) Extract the Square Root of $\left\{\begin{matrix} a^2 + 2\,a\,b + 2\,a\,c + b^2 \\ + 2\,b\,c + c^2 \end{matrix}\right\}$

44) $\sqrt{\left[9\,x^2 - 30\,a\,x - 3\,a^2\,x + 25\,a^2 + 5\,a^3 + \frac{a^4}{4}\right]}$
$=$ what ?

45) Extract the Square Root of $\left\{\begin{matrix} 4\,x^4 + 8\,a\,x^3 + 4\,a^2\,x^2 + 16\,b^2\,x^2 \\ + 16\,a\,b^2\,x + 16\,b^4 \end{matrix}\right\}$

46) $\sqrt{[9a^2 - 6ab + 30ac + 6ad + b^2 - 10bc - 2bd + 25c^2 + 10cd + d^2]}$
$=$ what ?

47) Extract the Square Root of $\frac{9}{4} + 6\,x - 17\,x^2 - 28\,x^3$
$+ 49\,x^4$

48) $\sqrt{\left[9\,x^4 - 3\,a\,x^3 + 6\,b\,x^3 + \frac{a^2\,x^2}{4} - a\,b\,x^2 + b^2\,x^2\right]}$
$=$ what ?

49) Extract the Root of $\frac{a^2 + 2\,a\,b + b^2}{x^2 - 2\,a\,x + a^2}$

50) $\sqrt{\frac{a^2 - 4\,a\,b + 4\,b^2}{x^4 + 6\,a^2\,x^2 + 9\,a^4}} =$ what ?

B. Cube Roots.

a. CUBE ROOTS OF NUMBERS.

1)	8	2)	64	3)	125
4)	343	5)	729	6)	1000
7)	1728	8)	13824	9)	12167
10)	884736	11)	405224	12)	166375
13)	1259712	14)	2460375	15)	11089567
16)	64000	17)	1191016	18)	17173512
19)	49836032	20)	40353607	21)	64481201
22)	8000000	23)	74088000		

24) $\sqrt[3]{318611987} =$ what ?

25) $\sqrt[3]{340068392} =$ what ?

26) $\sqrt[3]{63727838(4} =$ what ?

27) $\sqrt[3]{7256313856} =$ what ?

28) $\sqrt[3]{111980168000} =$ what ?

29) $\sqrt[3]{115145914625} =$ what ?

30) $\sqrt[3]{18970074963} =$ what ?

31) $\sqrt[3]{3108486729} =$ what ?

32) $\sqrt[3]{7} =$ what ? *

33) $\sqrt[3]{12} =$ what ?

34) $\sqrt[3]{82} =$ what ?

35) $\sqrt[3]{267} =$ what ?

36) $\sqrt[3]{551} =$ what ?

37) $\sqrt[3]{687} =$ what ?

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* Find 3 or 4 Decimals of the answer.

38) $\sqrt[3]{5.8}$ = what? 39) $\sqrt[3]{10.5}$ = what?

40) $\sqrt[3]{102.875}$ = what? 41) $\sqrt[3]{28.25}$ = what?

42) $\sqrt[3]{5.823}$ = what? 43) $\sqrt[3]{\frac{1}{8}}$ = what?

44) $\sqrt[3]{\frac{1}{27}}$ = what? 45) $\sqrt[3]{\frac{8}{81}}$ = what?

46) $\sqrt[3]{\frac{27}{64}}$ = what? 47) $\sqrt[3]{\frac{343}{512}}$ = what?

48) $\sqrt[3]{\frac{729}{125}}$ = what? 49) $\sqrt[3]{465\frac{31}{64}}$ = what?

50) $\sqrt[3]{52034\frac{10}{27}}$ = what? 51) $\sqrt[3]{\frac{2}{3}}$ = what?*

52) $\sqrt[3]{\frac{5}{6}}$ = what? 53) $\sqrt[3]{\frac{4}{5}}$ = what?

54) $\sqrt[3]{\frac{1}{2}}$ = what? 55) $\sqrt[3]{\frac{1}{3}}$ = what?

56) $\sqrt[3]{1\frac{1}{2}}$ = what? 57) $\sqrt[3]{3\frac{1}{4}}$ = what?

58) $\sqrt[3]{\frac{5}{14}}$ = what? 59) $\sqrt[3]{3\frac{4}{5}}$ = what?

60) $\sqrt[3]{15\frac{2}{3}}$ = what?

b. EXTRACTION OF THE CUBE ROOT OF ALGEBRAIC QUANTITIES.

1) Extract the Cube Root of b^3.
2) Extract the Cube Root of b^6.
3) Extract the Cube Root of a^9.
4) Extract the Cube Root of $-27\,a^3\,b^9$.
5) Extract the Cube Root of $125\,a^6\,b^3\,c^9$.
6) Extract the Cube Root of $343\,a^3\,b^6\,c^{12}$.

---

* Multiply first Numerator and Denominator by the square of the Denominator, or change the Vulgar Fraction into a Decimal.

7) Extract the Cube Root of $-\frac{a^3}{b^3}$.

8) Extract the Cube Root of $\frac{8\,a^3}{27\,c^6}$.

9) Extract the Cube Root of $\frac{1}{a^3}$.

10) Extract the Cube Root of $-\frac{1}{27\,a^3\,b^3}$.

11) Extract the Cube Root of $-\frac{1}{x^3\,y^3}$.

12) Extract the Cube Root of $a^3\,b^3\,c^9\,d^6$.

13) $\sqrt[3]{-\frac{a^6}{b^9}} =$ what ?

14) $\sqrt[3]{(a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3)} =$ what ?

15) $\sqrt[3]{(a^3 - 3\,a^2\,b + 3\,a\,b^2 - b^3)} =$ what ?

16) $\sqrt[3]{(x^3 + 6\,x^2 + 12\,x + 8)} =$ what ?

17) $\sqrt[3]{(x^3 + 3\,x^2 + 3\,x + 1)} =$ what ?

18) $\sqrt[3]{(8\,a^3 - 84\,a^2\,x + 294\,a\,x^2 - 343\,x^3)} =$ what ?

19) $\sqrt[3]{(x^6 - 6\,c\,x^5 + 12\,c^2\,x^4 - 8\,c^3\,x^3)} =$ what ?

20) $\sqrt[3]{(8 - 12\,x^5 + 6\,x^{10} - x^{15})} =$ what ?

21) $\sqrt[3]{\left(\frac{a^3}{b^3} + \frac{3\,a^2\,c}{b^2\,d} + \frac{3\,a\,c^2}{b\,d^2} + \frac{c^3}{d^3}\right)} =$ what ?

22) $\sqrt[3]{\left(\frac{1}{a^3} - \frac{3}{a^2\,b} + \frac{3}{a\,b^2} - \frac{1}{b^3}\right)}$ what ?

23) $\sqrt[3]{\left\{\begin{matrix} a^3+3a^2b+3a^2c+3ab^2+6abc \\ +3ac^2+b^3+3b^2c+3bc^2+c^3 \end{matrix}\right\}}$ = what?

24) $\sqrt[3]{(x^6+3x^5+6x^4+7x^3+6x^2+3x+1)}$ = what?

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c. HIGHER ROOTS OF SIMPLE QUANTITIES.*

1) Extract the Fourth Root of 16.†

2) Extract the Fourth Root of 81.

3) Extract the Fourth Root of 4096.

4) $\sqrt[4]{20736}$ = what?

5) $\sqrt[4]{390625}$ = what?

6) Extract the 6th Root of 4096. ‡

7) $\sqrt[6]{15625}$ = what?

8) $\sqrt[4]{256\,a^4\,b^8}$ = what?

9) $\sqrt[4]{\dfrac{a^8\,b^4}{81\,c^{12}}}$ = what?

10) $\sqrt[4]{\dfrac{1}{16a^8}}$ = what?

11) $\sqrt[6]{729\,a^{12}\,b^{18}\,c^6}$ = what?

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* For higher Roots of Compound Quantities see the Binomial Theorem.

† Extract first the *square* root of 16, and then again the square root of the answer.

‡ Extract first the square and then the cube root.

12) $\sqrt[6]{\frac{64\,a^{12}}{b^{18}\,c^{24}}} =$ what ?

D. Addition, Subtraction, Multiplication, and Division of Roots.

a. ADDITION.

1) $\sqrt{a} + \sqrt{a} =$ what ?
2) $\sqrt{2} + \sqrt{2} =$ what ?
3) $\sqrt{3} + 2\sqrt{3} + 4\sqrt{3} =$ what ?
4) $3\sqrt{b} + 2\sqrt{b} + \sqrt{b} =$ what ?
5) $6\sqrt{2} - 5\sqrt{2} + \sqrt{2} =$ what ?
6) $5\sqrt{5} - 3\sqrt{5} + \frac{3}{4}\sqrt{5} =$ what ?
7) $4\sqrt{3} - 2\sqrt{3} - \frac{1}{2}\sqrt{5} =$ what ?
8) $3\sqrt{\frac{3}{2}} - 2\sqrt{\frac{3}{2}} + \frac{1}{2}\sqrt{\frac{3}{2}} =$ what ?
9) $\sqrt[3]{a} + 2\sqrt[3]{a} =$ what ?
10) $3\sqrt[3]{a} + 2\sqrt[3]{a} - \frac{1}{2}\sqrt[3]{a} =$ what ?
11) $\sqrt[4]{a^3} - 2\sqrt[4]{a^3} + 5\sqrt[4]{a^3} =$ what ?
12) $\sqrt[3]{2} + 2\sqrt[3]{2} - \frac{1}{2}\sqrt[3]{2} =$ what?
13) $6\sqrt[3]{\frac{3}{2}} + 2\sqrt[3]{\frac{3}{2}} - \frac{3}{4}\sqrt[3]{\frac{3}{2}} =$ what ?
14) $2\sqrt[3]{3} + \sqrt[3]{3} + 4\sqrt{3} - \sqrt{3} =$ what ?
15) $\sqrt[5]{a} + \sqrt[5]{a} =$ what ?
16) $\sqrt[3]{4} + 3\sqrt[3]{4} - 2\sqrt[3]{3} - \sqrt[3]{3} =$ what ?
17) $\frac{1}{2}\sqrt{\frac{1}{2}} + \frac{1}{4}\sqrt{\frac{1}{2}} - \frac{1}{8}\sqrt{\frac{1}{2}} =$ what ?
18) $\frac{3}{4}\sqrt{2} + \frac{4}{5}\sqrt{2} - \frac{1}{2}\sqrt{2} =$ what ?

19) $\frac{5}{6}\sqrt[3]{7}-\frac{1}{2}\sqrt[3]{7}+\frac{2}{3}\sqrt[3]{7}=$ what ?

20) $\frac{5}{6}\sqrt[3]{9}-\frac{1}{2}\sqrt[3]{9}+\frac{3}{4}\sqrt{5}-\frac{1}{2}\sqrt{5}=$ what ?

21)
$$\text{Add}\left\{\begin{array}{rr}10\sqrt{2} & -\,5\sqrt[3]{7}\\ 5\sqrt{2} & +\,4\sqrt[3]{7}\\ \sqrt{2} & -\,3\sqrt[3]{7}\\ \hline\end{array}\right.$$

22)
$$\text{Add}\left\{\begin{array}{rrr}6\sqrt{2} & +\,5\sqrt{7} & +\ \sqrt[3]{6}\\ 3\sqrt{2} & -\,3\sqrt{7} & -\,4\sqrt[3]{6}\\ \sqrt{2} & -\ \sqrt{7} & -\,\frac{1}{2}\sqrt[3]{6}\\ 7\sqrt{2} & -\ \sqrt{7} & +\,\frac{7}{2}\sqrt[3]{6}\\ \hline\end{array}\right.$$

23)
$$\text{Add}\left\{\begin{array}{rrr}10\sqrt{2} & +\,5\sqrt[3]{6} & -\,2\sqrt[4]{5}\\ 5\sqrt{2} & +\ \sqrt[3]{6} & +\,4\sqrt[4]{5}\\ -\ 3\sqrt{2} & -\,9\sqrt[3]{6} & -\,3\sqrt[4]{5}\\ -\ \sqrt{2} & -\,2\sqrt[3]{6} & +\,6\sqrt[4]{5}\\ \hline\end{array}\right.$$

24)
$$\text{Add}\left\{\begin{array}{rrr}\sqrt{a} & +\,3\sqrt{a\,b} & -\,4\sqrt[3]{b}\\ 3\sqrt{a} & -\,2\sqrt{a\,b} & +\,2\sqrt[3]{b}\\ \frac{3}{2}\sqrt{a} & -\,\frac{1}{2}\sqrt{a\,b} & +\,\frac{3}{4}\sqrt[3]{b}\\ \frac{1}{4}\sqrt{a} & -\,\frac{3}{4}\sqrt{a\,b} & +\,\frac{1}{8}\sqrt[3]{b}\\ \hline\end{array}\right.$$

b. SUBTRACTION.

1) From $2\sqrt{3}$
Subtract $\underline{\sqrt{3}}$

2) From $4\sqrt{a}$
Subtract $\underline{\sqrt{a}}$

3) From $3\sqrt{a\,b}$
Subtract $\underline{\frac{1}{2}\sqrt{a\,b}}$

4) From $4\sqrt[3]{a}$
Subtract $\underline{2\sqrt[3]{a}}$

5) From $\frac{3}{4}\sqrt[3]{2}$
Subtract $\underline{\frac{1}{2}\sqrt[3]{2}}$

6) From $9\sqrt[3]{\frac{1}{2}}$
Subtract $\underline{5\sqrt[3]{\frac{1}{2}}}$

7) From $-4\sqrt{2}$
Subtract $\underline{3\sqrt{2}}$

8) From $3\sqrt{3}$
Subtract $\underline{-2\sqrt{3}}$

9) From $-3\sqrt{2}$
Subtract $\underline{-2\sqrt{2}}$

10) From $-\sqrt[3]{3}$
Subtract $\underline{-\frac{1}{2}\sqrt[3]{3}}$

11) From $18\sqrt{7}-5\sqrt{6}+10\sqrt[3]{11}$
Subtract $\underline{6\sqrt{7}-2\sqrt{6}+\frac{17}{2}\sqrt[3]{11}}$

12) From $4\sqrt{6}-3\sqrt[3]{2}+\sqrt[4]{5}$
Subtract $\underline{-3\sqrt{6}+2\sqrt[3]{2}-\frac{1}{2}\sqrt[4]{5}}$

13) From $3\sqrt{a}+5\sqrt{b}-\sqrt[3]{c}$
Subtract $\underline{2\sqrt{a}-3\sqrt{b}-\sqrt[3]{c}}$

14) From $4\sqrt{ab}-\sqrt{c}+\sqrt{bc}$
Subtract $\underline{-3\sqrt{ab}-\sqrt{c}+\frac{3}{4}\sqrt{bc}}$

15) From $2\sqrt[4]{a}+3\sqrt[3]{b}-4\sqrt[3]{c}$
Subtract $\underline{\sqrt[4]{a}-2\sqrt[3]{b}-4\sqrt[3]{c}}$

16) From $3\sqrt[4]{a^3}+5\sqrt[3]{a^2b}-6\sqrt{c}$
Subtract $\underline{-2\sqrt[4]{a^3}+3\sqrt[3]{a^2b}+3\sqrt{c}}$

c. ABBREVIATIONS AND TRANSFORMATIONS. *

The root of a product is obtained by taking the root of its factors. This leads to several abbreviations and transformations. The square root of 24, for instance, is equal to the square root of 4 multiplied by the square root of 6; because 4 and 6 are the factors of 24; which may be expressed thus: $\sqrt{24} = \sqrt{4}.\sqrt{6}$. But $\sqrt{4} = 2$; therefore $\sqrt{4} \times \sqrt{6} = 2 \times \sqrt{6} = 2\sqrt{6}$. On the same principle may the following expressions be simplified.

1) $\left.\begin{array}{l}\text{If } \sqrt{24} = \sqrt{4}.\sqrt{6} \\ \text{and } \sqrt{54} = \sqrt{9}.\sqrt{6}\end{array}\right\}$ what are $\sqrt{24} + \sqrt{54}$ equal to?

2) $\left.\begin{array}{l}\text{If } 2\sqrt{8} = \sqrt{4}.\sqrt{2} \\ \text{and } 3\sqrt{18} = \sqrt{9}.\sqrt{2}\end{array}\right\}$ what are $2\sqrt{8} + 3\sqrt{18}$ equal to?

3) $\left.\begin{array}{l}\text{If } \sqrt{12} = \sqrt{4}.\sqrt{3} \\ \text{and } \sqrt{27} = \sqrt{9}.\sqrt{3}\end{array}\right\}$ what are $\sqrt{12} + \sqrt{27}$ equal to?

4) $5\sqrt{8} - 4\sqrt{50} =$ what?

5) $\left.\begin{array}{l}\text{If } \sqrt{\frac{3}{4}} = \sqrt{\frac{1}{4}}.\sqrt{3} \\ \text{and } \sqrt{12} = \sqrt{4}.\sqrt{3}\end{array}\right\}$ what are $\sqrt{\frac{3}{4}} + \sqrt{12}$ equal to?

6) $3\sqrt{\frac{3}{4}} + 2\sqrt{12} =$ what?

7) $\left.\begin{array}{l}\text{If } \sqrt[3]{54} = \sqrt[3]{27}.\sqrt[3]{2} \\ \text{and } \sqrt[3]{16} = \sqrt[3]{8}.\sqrt[3]{2}\end{array}\right\}$ what are $2\sqrt[3]{54} - 7\sqrt[3]{16}$ equal to?

8) $5\sqrt[3]{54} + 3\sqrt[3]{16} =$ what?

9) If $\sqrt[3]{128} = \sqrt[3]{64}.\sqrt[3]{2}$, what are $\sqrt[3]{128} + \sqrt[3]{2}$ equal to?

10) If $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{4}.2} = \sqrt{\frac{1}{4}}.\sqrt{2}$, what are $\sqrt{\frac{1}{2}} + \sqrt{2}$ equal to?

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* If the pupil should find these problems too difficult, they may be omitted till reviewing the book, or until the pupil is further advanced.

11) $3\sqrt{\tfrac{1}{2}}+\sqrt{2}=$ what?

12) $\tfrac{1}{2}\sqrt{\tfrac{1}{2}}+\tfrac{3}{4}\sqrt{2}=$ what?

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d. MULTIPLICATION OF ROOTS.

1. MULTIPLICATION OF SIMPLE QUANTITIES.

1) Multiply $\sqrt{2}$, by $\sqrt{2}$.
2) Multiply $\sqrt{3}$, by $\sqrt{3}$.
3) Multiply $\sqrt{5}$, by $\sqrt{2}$.
4) Multiply $\sqrt{6}$, by $2\sqrt{5}$.
5) Multiply $3\sqrt{6}$, by $2\sqrt{5}$.
6) Multiply $14\sqrt{2}$, by $3\sqrt{6}$.
7) Multiply $\sqrt{a}$, by $\sqrt{a}$.
8) Multiply $\sqrt{b}$, by $\sqrt{b}$.
9) Multiply $\sqrt{a}$, by $\sqrt{b}$.
10) Multiply $\sqrt{a}$, by $b\sqrt{c}$.
11) Multiply $a\sqrt{b}$, by $c\sqrt{d}$.
12) Multiply $c\sqrt{a}$, by $d\sqrt{a}$.
13) Multiply $\sqrt[3]{a}$, by $\sqrt[3]{a^2}$.
14) Multiply $\sqrt[3]{b^2}$, by $\sqrt[3]{b}$.
15) Multiply $\sqrt[3]{a}$, by $\sqrt[3]{b}$.
16) Multiply $a\sqrt[3]{b}$, by c.

17) $a\sqrt[3]{b}\times c\sqrt[3]{d}=$ what?

18) $3a\sqrt[3]{b^2}\times 5b\sqrt[3]{b}=$ what?

19) $5\sqrt[3]{12}\times 3\sqrt[3]{6}=$ what?

20) $3\sqrt[3]{4}\times 4=$ what?

21) $15\times 3\sqrt{3}=$ what?

22) $\sqrt{\tfrac{2}{3}}\times\sqrt{\tfrac{4}{5}}=$ what?

23) $\sqrt{\tfrac{3}{4}}\times\sqrt{5}=$ what?

24) $3\sqrt{\tfrac{3}{8}}\times\sqrt{8}=$ what?

25) $4\sqrt{\tfrac{1}{2}}\times 3\sqrt{\tfrac{2}{3}}=$ what?

26) $\tfrac{1}{2}\sqrt{\tfrac{2}{3}}\times\tfrac{3}{4}\sqrt{\tfrac{4}{5}}=$ what?

27) $\dfrac{a}{b}\sqrt{\dfrac{c}{d}}\times\dfrac{e}{f}\sqrt{\dfrac{g}{h}}=$ what?

28) $\sqrt{\frac{1}{a}} \times \sqrt{\frac{1}{b}} =$ what?

29) $\sqrt{\frac{1}{a}} \times \sqrt{a} =$ what?

30) $\sqrt{\frac{1}{3}} \times \sqrt{3} =$ what?

31) $3\sqrt{\frac{1}{3}} \times 4\sqrt{3} =$ what?

32) $\frac{1}{a}\sqrt{\frac{1}{b}} \times \frac{1}{c}\sqrt{\frac{1}{d}}$ what?

33) $\frac{1}{2}\sqrt{\frac{1}{3}} \times \frac{1}{3}\sqrt{\frac{1}{4}} =$ what?

34) $\frac{1}{2}\sqrt{\frac{1}{3}} \times \frac{1}{4}\sqrt{\frac{1}{5}} =$ what?

35) $\sqrt[3]{\frac{a}{b}} \times \sqrt[3]{\frac{a}{b}}$ what?

36) $\sqrt[3]{\frac{a}{b}} \times \sqrt[3]{\frac{c}{d}} =$ what?

37) $a\sqrt[3]{\frac{b}{c}} \times d\sqrt[3]{\frac{e}{f}} =$ what?

38) $\frac{a}{b}\sqrt[3]{\frac{c}{d}} \times \frac{e}{f}\sqrt[3]{\frac{g}{h}} =$ what?

39) $\sqrt[3]{\frac{1}{a}} \times \sqrt[3]{\frac{1}{b}} =$ what?

40) $\sqrt[3]{\frac{1}{a}} \times \sqrt[3]{a^2} =$ what?

41) $a\sqrt[3]{\frac{1}{b}} \times \sqrt[3]{b^4} =$ what?

42) $a\sqrt[3]{\frac{1}{b}} \times c\sqrt[3]{\frac{1}{d}} =$ what?

43) $\frac{1}{a}\sqrt[3]{\frac{1}{b}} \times \frac{1}{d}\sqrt[3]{\frac{1}{f}} =$ what?

44) $\sqrt[3]{\frac{1}{2}} \times \sqrt[3]{\frac{1}{3}} =$ what?

45) $\sqrt[3]{\frac{1}{16}} \times \sqrt[3]{2} =$ what?

46) $5\sqrt[3]{\frac{1}{81}} \times 3\sqrt[3]{3} =$ what?

47) $\sqrt[4]{a} \times \sqrt[4]{a} =$ what?

48) $\sqrt[4]{a\,b^2} \times \sqrt[4]{c} =$ what?

49) $4\sqrt[4]{15} \times 3\sqrt[4]{2} =$ what?

50) $\frac{1}{4}\sqrt[4]{6} \times \frac{1}{2}\sqrt[4]{3} =$ what?

51) $\sqrt{a\,b} \times \sqrt{c\,d} =$ what?

52) $a\sqrt{b\,c\,d} \times e\sqrt{b^3\,c} =$ what?

53) $3\sqrt[3]{a^2\,b} \times 4\sqrt[3]{a\,b^2} =$ what?

54) $4\sqrt[3]{a^2\,c} \times \sqrt[3]{b\,c} =$ what?

55) $\sqrt[3]{a\,b\,c^2} \times \sqrt[3]{d\,e^2 f} =$ what?

56) $\sqrt[3]{\frac{1}{a^2\,b}} \times \sqrt[3]{\frac{a^2}{b^2}} =$ what?

57) $\sqrt[3]{\frac{1}{c^2 f^2}} \times 3\sqrt[3]{\frac{c^2}{f}} =$ what?

58) $\sqrt{\frac{1}{f}} \times \sqrt{\frac{1}{c\,f}} =$ what?

59) $\frac{1}{a}\sqrt{\frac{1}{b}} \times a\sqrt{\frac{1}{b}} =$ what ?

60) $3\sqrt{\tfrac{1}{3}} \times \tfrac{1}{3}\sqrt{3} =$ what ?

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2. MULTIPLICATION OF COMPOUND QUANTITIES.

1) Multiply $3 + \sqrt{5}$
by $\sqrt{2}$

2) Multiply $3\sqrt{2} + 4\sqrt{5}$
by 2

3) Multiply $3\sqrt{2} + 4\sqrt{5}$
by $\sqrt{2}$

4) Multiply $3 + \sqrt{5} - 2\sqrt{3}$
by $\sqrt{3}$

5) Multiply $3 + \sqrt{5}$
by $2 - \sqrt{5}$

6) Multiply $2 + \sqrt{3}$
by $1 - \sqrt{3}$

7) Multiply $7 + 2\sqrt{6}$
by $9 - 5\sqrt{6}$

8) Multiply $3 - 2\sqrt{6}$
by $2 - 3\sqrt{6}$

9) Multiply $1 - \sqrt{2}$
by $1 - \sqrt{2}$

10) Multiply $6 + 3\sqrt{7}$
by $3 - \sqrt{7}$

11) Multiply $-5 - \sqrt{\tfrac{1}{2}}$
by $-5 + \sqrt{\tfrac{1}{2}}$

12) Multiply $9 + 2\sqrt{10}$
by $9 - 2\sqrt{10}$

13) Multiply $\sqrt{2} + \sqrt{3}$
by $2\sqrt{2} - \sqrt{3}$

14) Multiply $5\sqrt{3} + 3\sqrt{5}$
by $7\sqrt{3} - 2\sqrt{5}$

15) Multiply $\sqrt{2} + \sqrt{3}$
by $\sqrt{2} + \sqrt{3}$

16) Multiply $a + \sqrt{b}$
by $a - \sqrt{b}$

17) Multiply $\sqrt{a} + \sqrt{b}$
by $\sqrt{a} + \sqrt{b}$

18) Multiply $c\sqrt{a} + a\sqrt{b}$
by $c\sqrt{a} - d\sqrt{b}$

19) Multiply $3\sqrt{2} + 4\sqrt{5}$
by $3\sqrt{2} - 4\sqrt{5}$

20) Multiply $3\sqrt{7} + 2\sqrt{6}$
by $3\sqrt{7} + 2\sqrt{6}$

21) Multiply $3\sqrt{7}-2\sqrt{6}$
by $\underline{3\sqrt{7}-2\sqrt{6}}$

22) Multiply $\sqrt[3]{2}+2\sqrt[3]{3}$
by $\underline{\sqrt[3]{2}-2\sqrt[3]{3}}$

23) Multiply $\sqrt[3]{\tfrac{1}{2}}+\sqrt[3]{\tfrac{1}{3}}$
by $\underline{\sqrt[3]{\tfrac{1}{2}}-\sqrt[3]{\tfrac{1}{3}}}$

24) Multiply $\sqrt{\frac{a}{b}}+\sqrt{\frac{c}{d}}$
by $\underline{\sqrt{\frac{a}{b}}-\sqrt{\frac{c}{d}}}$

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e. DIVISION.

1) Divide $\sqrt{a}$ by $\sqrt{a}$.
2) Divide $\sqrt{a^3}$ by $\sqrt{a}$.
3) Divide $\sqrt{a}$ by $\sqrt{b}$.
4) Divide $4\sqrt{a^3}$ by $2\sqrt{a}$.
5) Divide $6\sqrt{a^3}$ by $2\sqrt{a}$.
6) Divide $4\sqrt{a}$ by $2\sqrt{b}$.
7) Divide $3\sqrt{b}$ by $4\sqrt{c}$.
8) Divide $a\sqrt{b}$ by $c\sqrt{d}$.
9) Divide $\sqrt{a^2}$ by $\sqrt{a}$.
10) Divide a by $\sqrt{a}$.
11) Divide $4\sqrt{a^2\,b^3}$ by $2\sqrt{a\,b}$.
12) Divide $a\,b$ by $\sqrt{a\,b}$.
13) Divide $\sqrt{\frac{a}{b}}$ by $\sqrt{\frac{c}{d}}$
14) Divide $4\sqrt{\frac{a}{b}}$ by $2\sqrt{\frac{c}{d}}$
15) Divide $\frac{m}{n}\sqrt{\frac{a}{b}}$ by $\frac{p}{q}\sqrt{\frac{c}{d}}$
16) Divide $\sqrt{\frac{1}{a}}$ by $\sqrt{\frac{1}{b}}$

17) Divide $\sqrt{\frac{1}{2}}$ by $\sqrt{\frac{1}{3}}$. 18) Divide $\sqrt{\frac{2}{3}}$ by $\sqrt{\frac{4}{5}}$.
19) Divide $\frac{1}{2}\sqrt{\frac{2}{3}}$ by $\frac{5}{6}\sqrt{\frac{4}{5}}$. 20) Divide $\frac{2}{3}\sqrt{\frac{1}{2}}$ by $\frac{4}{5}\sqrt{\frac{1}{3}}$.

21) $\sqrt[3]{a^2} : \sqrt[3]{a^2} =$ what? 22) $\sqrt[3]{a^2} : \sqrt[3]{a} =$ what?
23) $\sqrt[3]{a} : \sqrt[3]{b} =$ what? 24) $4\sqrt[3]{a^2} : 2\sqrt[3]{a} =$ what?
25) $8\sqrt[3]{a^2} : 4\sqrt[3]{a} =$ what? 26) $10\sqrt[3]{a^2} : 5\sqrt[3]{b} =$ what?
27) $2\sqrt[3]{b} : 5\sqrt[3]{c} =$ what? 28) $a\sqrt[3]{b} : c\sqrt[3]{d} =$ what?
29) $\sqrt[3]{a^3} : \sqrt[3]{a} =$ what? 30) $a : \sqrt[3]{a} =$ what?

31) $6\sqrt[3]{a^3 b^2} : 2\sqrt[3]{a b} =$ what?

32) $a b : \sqrt[3]{a b} =$ what?

33) $\sqrt[3]{\frac{a}{b}} : \sqrt[3]{\frac{c}{d}} =$ what?

34) $4\sqrt[3]{\frac{a}{b}} : 2\sqrt[3]{\frac{c}{d}} =$ what?

35) $\frac{m}{n}\sqrt[3]{\frac{a}{b}} : \frac{p}{q}\sqrt[3]{\frac{c}{d}} =$ what?

36) $\sqrt[3]{\frac{1}{a}} : \sqrt[3]{\frac{1}{b}} =$ what?

37) $\sqrt[3]{\frac{1}{2}} : \sqrt[3]{\frac{1}{3}} =$ what?

38) $\sqrt[3]{\frac{2}{3}} : \sqrt[3]{\frac{4}{5}} =$ what?

39) $\frac{1}{2}\sqrt[3]{\frac{2}{3}} : \frac{5}{6}\sqrt[3]{\frac{4}{5}} =$ what?

40) $\frac{2}{3}\sqrt[3]{\frac{1}{2}} : \frac{4}{5}\sqrt[3]{\frac{1}{3}} =$ what?

E. Fractional Exponents.

$$\left.\begin{array}{l} 1)\ \text{Write } \sqrt{a} \\ 2)\ \text{Write } \sqrt{a^2} \\ 3)\ \text{Write } \sqrt[3]{a^2} \\ 4)\ \text{Write } \sqrt[4]{a^3} \\ 5)\ \text{Write } \sqrt[5]{a^3} \\ 6)\ \text{Write } \sqrt[7]{a^3} \\ 7)\ \text{Write } \sqrt[3]{a} \\ 8)\ \text{Write } \sqrt[4]{a} \end{array}\right\}\ \text{with fractional Exponents.}$$

9) $\sqrt{a} \times \sqrt[3]{a} = a^{\frac{1}{2}} \times a^{\frac{1}{2}} =$ what ? *

Write $a^{\frac{1}{6}}$ in form of a root.

10) $\sqrt{a} \times \sqrt[4]{a} = a^{\frac{1}{2}} \times a^{\frac{1}{4}} =$ what ?

Write the answer in form of a root.

11) $\sqrt{a} \times \sqrt[3]{a^2} = a^{\frac{1}{2}} \times a^{\frac{2}{3}} =$ what ?

Write the answer in form of a root ?

12) $\sqrt[3]{a^2} \times \sqrt[4]{a^3} = a^{\frac{2}{3}} \times a^{\frac{3}{4}} =$ what ?

Write the answer in form of a root.

13) $\sqrt[5]{a^2} \times \sqrt[2]{a} =$ what ?

Write the answer in form of a root.

14) $\sqrt[3]{a} \times \sqrt{a^3} =$ what ?

Write the answer in form of a root.

15) $\sqrt[4]{a^3} \times \sqrt[3]{a^2} =$ what ?

Write the answer in form of a root.

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* Add the exponents ½, ⅓, after having reduced them to a common Denominator.

16) $\sqrt[6]{a^3} \times \sqrt[4]{a^2} =$ what ?
Write the answer in form of a root.

17) $\sqrt{a} \times \sqrt[6]{a^5} =$ what ?
Write the answer in form of a root.

18) $\sqrt[3]{a^2} \times \sqrt[6]{a^5} =$ what ?
Write the answer in form of a root.

19) $\sqrt[3]{a^2} : \sqrt{a} = a^{\frac{2}{3}} : a^{\frac{1}{2}} = a^{\frac{4}{6}} : a^{\frac{3}{6}} =$ what ?
Write the answer in form of a root.

20) $\sqrt[4]{a^3} : \sqrt[3]{a^2} = a^{\frac{3}{4}} : a^{\frac{2}{3}} = a^{\frac{9}{12}} : a^{\frac{8}{12}} =$ what ?
Write the answer in form of a root.

21) $\sqrt{a} : \sqrt[3]{a} = a^{\frac{1}{2}} : a^{\frac{1}{3}} =$ what ?
Write the answer in form of a root.

22) $\sqrt[3]{a} : \sqrt[4]{a} = a^{\frac{1}{3}} : a^{\frac{1}{4}} =$ what ?
Write the answer in form of a root.

23) $\sqrt[3]{a^2} : \sqrt[5]{a^3} = a^{\frac{2}{3}} : a^{\frac{3}{5}} =$ what ?
Write the answer in form of a root.

24) $a^{\frac{7}{4}}\ a^{\frac{5}{6}} : =$ what ?
Write the answer in form of a root.

25) Write $\sqrt[3]{a^2\, b\, c^4} = \sqrt[3]{a^2}.\ \sqrt[3]{b}.\ \sqrt[3]{c^4}$ without a radical sign.

26) Write $\sqrt[4]{a^3\, b^2\, c} = \sqrt[4]{a^3} \times \sqrt[4]{b^2} \times \sqrt[4]{c^1}$ without a radical sign.

27) Write $\sqrt[3]{a^2\, c}$ without a radical sign.

28) Write $\sqrt[4]{a^3\, c^2}$ without a radical sign.

29) Multiply $\sqrt[4]{a^3\, b^2}$, by $\sqrt[2]{a\, b}$,
or which is the same $a^{\frac{3}{4}}\ b^{\frac{2}{4}}$, by $a^{\frac{1}{2}}\ b^{\frac{1}{2}}$.

30) Multiply $\sqrt[3]{a^2 b}$, by $\sqrt[4]{a^3 b^2}$,
or $a^{\frac{2}{3}} b^{\frac{1}{3}}$, by $a^{\frac{3}{4}} b^{\frac{2}{4}}$, and write the answer in form of a root.

31) Divide $a^{\frac{3}{4}} b^{\frac{1}{2}}$, by $a^{\frac{1}{2}} b^{\frac{1}{4}}$.
Write the answer in form of a root.

32) $\sqrt[3]{a^2 b^5} : \sqrt{a b^3} =$ what?
Write the answer in form of a root.

F. Powers and Roots of Roots.

1) What is the square of the Square Root of a?

2) What is the square of the Square Root of b?

3) What is the square of $\sqrt{a b}$?

4) What is the square of $\sqrt{2}$?

5) What is the square of $\sqrt{5}$?

6) $(\sqrt{3})^2 =$ what?

7) $(\sqrt{b})^3 = (b^{\frac{1}{2}})^3 =$ what?*
Write the answer in form of a root.

8) $(\sqrt{a^3})^4 = (a^{\frac{3}{2}})^4 =$ what?
Write the answer in form of a root.

9) $(\sqrt{a^3})^3 = (a^{\frac{3}{2}})^3 =$ what?
Write the answer in form of a root.

10) $(\sqrt{a^3})^5 =$ what?

11) $(\sqrt[3]{a^2})^2 =$ what?

12) $(\sqrt[3]{a^4})^3 =$ what?

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* Multiply the exponent ½ by 3.

13) $\sqrt{\sqrt{a}} = \sqrt{a^{\frac{1}{2}}} = (a^{\frac{1}{2}})^{\frac{1}{2}} =$ what ? *
Write the answer in form of a root.

14) $\sqrt[3]{\sqrt{a}} = \sqrt[3]{a^{\frac{1}{2}}} = (a^{\frac{1}{2}})^{\frac{1}{3}} =$ what?
Write the answer in form of a root.

15) $\sqrt{\sqrt{a^3}} = \sqrt{a^{\frac{3}{2}}} = (a^{\frac{3}{2}})^{\frac{1}{2}} =$ what ?
Write the answer in form of a root.

16) $\sqrt[4]{\sqrt{a^5}} = \sqrt[4]{a^{\frac{5}{2}}} = (a^{\frac{1}{4}})^{\frac{1}{3}} =$ what ?
Write the answer in form of a root.

17) $\sqrt[3]{\sqrt[4]{a}} = \sqrt[3]{a^{\frac{1}{4}}} = (a^{\frac{1}{4}})^{\frac{1}{3}} =$ what ?
Write the answer in form of a root.

18) $\sqrt{\sqrt[3]{a^2}} = \sqrt{a^{\frac{2}{3}}} = (a^{\frac{2}{3}})^{\frac{1}{2}} =$ what ?
Write the answer in form of a root.

19) $\sqrt{\sqrt{a^3}} =$ what ?
20) $\sqrt[3]{\sqrt{a^7}} =$ what ?
21) $\sqrt{\sqrt[3]{a^7}} =$ what ?
22) $\sqrt[4]{\sqrt{a}} =$ what ?
23) $\sqrt{\sqrt[4]{a}} =$ what ?
24) $\sqrt{\sqrt{\sqrt{a}}} =$ what ?

G. *Imaginary Quantities,*

Arising from the Impossibility of Extracting an Even Root out of a Negative Quantity.

[These problems are intended for pupils who are a little farther advanced, and may be omitted by very young ones]

ADDITION.

1) Add $\sqrt{-1}$, $2\sqrt{-1}$, $3\sqrt{-1}$.

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* Multiply the exponent ½ by itself.

2) $4\sqrt{-1} + \sqrt{-1} - 2\sqrt{-1} =$ what?

3) Add $\left\{\begin{array}{l} 3\sqrt{-1} + \sqrt{3} \\ 4\sqrt{-1} - 2\sqrt{3} \end{array}\right.$

4) Add $\left\{\begin{array}{l} 4\sqrt{-1} + 3\sqrt{-4} \\ 3\sqrt{-1} - 2\sqrt{-4} \end{array}\right.$

5) Add $\left\{\begin{array}{l} 3\sqrt{-1} + 4\sqrt{-6} - \sqrt{-5} \\ 2\sqrt{-1} - 3\sqrt{-6} + \sqrt{-5} \end{array}\right.$

6) Add $\left\{\begin{array}{r} 4\sqrt{-1} + 3 \\ 3\sqrt{-1} + 1 \\ -7\sqrt{-1} - 4 \end{array}\right.$

SUBTRACTION.

1) From $\sqrt{-1}$
Subtract $\sqrt{-1}$

2) From $5\sqrt{-1}$
Subtract $-2\sqrt{-1}$

3) From $-4\sqrt{-1}$
Subtract $3\sqrt{-1}$

4) From $-4\sqrt{-1}$
Subtract $-3\sqrt{-1}$

5) From $14\sqrt{-1} + 10$
Subtract $13\sqrt{-1} + 9$

6) From $3\sqrt{-1} + 2\sqrt{-3} - 8$
Subtract $2\sqrt{-1} - \sqrt{-3} + 8$

MULTIPLICATION.

1) $\sqrt{-1} \times \sqrt{-1} =$ what?

(The pupil ought to recollect that $\sqrt{-1}$ is a quantity which, multiplied by itself, produces -1.)

2) $3 \times \sqrt{-1} =$ what?

3) $a \times b\sqrt{-1} =$ what?

4) $2\sqrt{-1} \times 3\sqrt{-1} =$ what? *

5) $\sqrt{-a} \times \sqrt{-b} = \sqrt{a.-1} \times \sqrt{b.-1}$
$= \sqrt{a}\sqrt{-1} \times \sqrt{b}\sqrt{-1} =$ what? †

6) $c\sqrt{-a} \times d\sqrt{-b} =$ what?

DIVISION.

1) $\sqrt{-1} : \sqrt{-1} =$ what?
(How often is everything contained in itself?)

2) $b\sqrt{-1} : c\sqrt{-1} =$ what?
(Divide b by c, and $\sqrt{-1}$ by $\sqrt{-1}$, as in problem 1.)

---

* The pupil ought to recollect that $\sqrt{-1} \times \sqrt{-1} = -1$.

† $-a$ may be considered as a product of a multiplied by -1; and as the square root of a product is taken by taking the root of each factor, $\sqrt{-a} = \sqrt{a}.\sqrt{-1}$.

3) $1 : \sqrt{-1} = \frac{-1 \times -1}{\sqrt{-1}} = \frac{-\sqrt{-1}.\sqrt{-1}}{\sqrt{-1}}$

$$= \frac{-\sqrt{-1}\sqrt{-1}}{\sqrt{-1}} = \text{what?}\ ^*$$

(Reduce this fraction to its lowest term.)

4) $a : b\sqrt{-1} = a.\ 1 : b.\sqrt{-1} =$ what?
(Divide a by b, and 1 by $\sqrt{-1}$, as in the preceding problem.)

5) $4 : 2\sqrt{-1} =$ what?

6) $4\sqrt{-1} : 2\sqrt{-1} =$ what?

---

* $+1$ is equal to -1 multiplied by -1; but -1 itself is a product of $\sqrt{-1}$ multiplied by $\sqrt{-1}$; wherefore -1×-1 is also equal to $-1 \times \sqrt{-1}.\sqrt{-1}$, which when divided by the factor $\sqrt{-1}$, gives $-\sqrt{-1}$ for the answer.

Remark. In some treatises on Algebra $1 : \sqrt{-1}$ is put down as equal to $\sqrt{-1}$; and the reason alleged for it is, because $1 : \sqrt{-1}$, may be written $\sqrt{1} : \sqrt{-1} = \sqrt{\frac{1}{-1}} = \sqrt{-1}$. This reasoning however, is erroneous; but it is an error which *Euler* himself committed, and the same mistake has been preserved in three American versions of a treatise on Algebra by the same Author. It ought to be recollected that it is absurd to take an *imaginary* factor $\sqrt{-1}$ out of a product composed only of *real* factors. Moreover we may easily detect the fallacy of this reasoning, in the folowing manner:

$$\text{If } \frac{1}{\sqrt{-1}} = \sqrt{-1};$$

$$\text{then } \sqrt{-1} \times \sqrt{-1} \text{ must give } +1;$$

because the divisor multiplied by the quotient must reproduce the dividend. But $\sqrt{-1} \times \sqrt{-1} = -1$; as we know from the very definition of imaginary quantities, (and as is admitted by Euler himself in the same treatise;) consequently $\sqrt{-1}$ is the quotient of $-1 : \sqrt{-1}$, and not of $+1 : \sqrt{-1}$; which must be evidently of a contrary sign.

Besides $-\sqrt{-1} \times \sqrt{-1} = -1.\sqrt{-1} \times 1.\sqrt{-1} = -1 \times -1$, actually reproduces the dividend $+1$ of the division $+1 : \sqrt{-1}$.

SECTION V.

OF LOGARITHMS.

1) How do you take the Logarithm of the product of $A\,B$?

2) How do you take the Logarithm of a fraction $\frac{A}{B}$?

3) How do you take the Logarithm of a power A^3?

4) How do you take the Logarithm of a root $\sqrt[3]{A}$?

APPLICATIONS.

1) What is the log of (93×3514)?
2) What is the log of (1225×387)?
3) Take the log of (628×493).
4) Take the log of $(3748 \times 1752 \times 4065)$.
5) Take the log of $\frac{5}{4}$. (Subtract log 4 from log 5.)
6) Take the log of $\frac{39}{7}$.
7) Take the log of $\frac{14}{3}$.
8) Take the log of $15\frac{3}{4}$. $=$ log of $\frac{63}{4}$.
9) Take the log of $7\frac{3}{14}$. (Proceed as in the last problem.)
10) Take the log of $367\frac{5}{8}$.
11) Take the log of $187\frac{9}{11}$.
12) Take the log of $\frac{2}{3}$. (Add 1 to the index of log 2, and annex -1 at the end.)
13) Take the log of $\frac{5}{8}$. (Proceed as in problem 12.)
14) Take the log of $\frac{1}{9}$.

15) Take the log of $\frac{1}{20}$? (Add 2 to the index of log 1 = 0, and annex — 2 at the end.)

16) Take the log of $\frac{6}{37}$?

17) Take the log of $\frac{357}{695}$?

18) Take the log of $\frac{3542}{7965}$?

19) Take the log of 3 . 5?

20) Take the log of 12 . 63?

21) Take the log of 1. 3567?

22) Take the log of 0 . 7? (Take the log of 7, and add — 1 at the end.)

23) Take the log of 0 . 036? (Take the log 36 and add — 2 at the end?

24) Take the log of 0. 0065? (Take the log 65 and add — 3 at the end.)

25) Take log 0,0005637? (Take log 5637 and add — 4 at the end.

26) Take log $\frac{319 \times 7.655}{138}$ (Add the log of 319 and 7.655, and subtract log 138.)

27) Take log $\frac{213 \times 7.655}{3145 \times 718}$ (Add the log of 213 and log 7.655 and subtract the logarithms of 3145 and 718.

28) Take log $\frac{24 \times 47.34}{7.5 \times 6.3}$?

29) Take log 3^{15}? (Multiply log 3 by 15.)

30) Take log 5^{27}? (Multiply log 5 by 27.)

31) Take log 16^{20}?

32) Take log $(\frac{5}{4})^{12}$? (Take log $\frac{5}{4}$ as in problem 5, and multiply it by 12.

33) Take log $(\frac{7}{3})^{14}$?

34) Take log $(\frac{15}{13})^{16}$?

35) Take log $(\frac{12}{5})^{32}$?

36) Take log $(\frac{2}{3})^{15}$? (Take log $\frac{2}{3}$ as in problem 12, and multiply the whole by 15.)

37) Take log $(\frac{5}{8})^{14}$?

38) Take log $(\frac{3}{4})^{30}$?

39) Take log $\sqrt{5}$. (Take log 5 and divide it by 2.)

40) Take log $\sqrt{135}$. 41) Take log $\sqrt[3]{135}$.

42) Take log $\sqrt[8]{15276}$. 43) Take log $\sqrt[5]{35107}$.

44) Take log $\sqrt[100]{13}$.

45) Take log $\sqrt[5]{\frac{14}{3}}$. (Take log $\frac{14}{3}$ as in problem 7, and divide by 7.)

46) Take log $\sqrt[7]{\frac{15}{4}}$?

47) Take log $\sqrt{\frac{4}{9}}$?

48) Take log $\sqrt[11]{(\frac{12}{7})^{28}}$. (Take log $\frac{12}{7}$, multiply it by 28 and divide the answer by 11.)

ACTUAL CALCULATIONS OF SOME NUMERICAL EXPRESSIONS BY MEANS OF LOGARITHMS.

1) $\sqrt[7]{8}$ = what? (Take log 8, divide it by 7, and take the corresponding number to the next lower logarithm in the tables. In the same manner solve the following problems.

2) $\sqrt[4]{35246}$ = what? 3) $\sqrt[12]{5673}$ = what?

4) $\sqrt[6]{235.7}$ = what? 5) $\sqrt[5]{\frac{13}{16}}$ = what?

6) $\sqrt[7]{\frac{1171}{345}}$ = what? 7) $\sqrt[8]{172\frac{5}{6}}$ = what?

8) $\sqrt[13]{\frac{3348}{569}}$ = what?

9) $(4)^{15}$ = what? (Take log 4, multiply it by 15, and take the corresponding number to the next lower logarithm in the book.)

10) $(6)^{12}$ = what? 11) $(\frac{9}{8})^{21}$ = what?

12) $(\frac{17}{6})^{9}$ = what? 13) $(\frac{643}{637})^{123}$ = what?

14) $(317\frac{3}{4})^{0.6}$ = what? (Take log $\frac{1271}{4}$, multiply by 0.6, and proceed as with other problems.

15) $(\frac{167}{53})^{0.32}$ = what? 16) $(\frac{1}{2})^{15}$ = what? *

17) $(\frac{1}{4})^{16}$ = what? 18) $(\frac{3}{4})^{10}$ = what?

19) $(\frac{5}{6})^{8}$ = what? 20) $(\frac{6}{5})^{8}$ = what?

21) $(\frac{4}{3})^{10}$ = what? 22) $(\frac{7}{8})^{6}$ = what?

23) $(\frac{8}{7})^{6}$ = what? 24) $(\frac{10}{9})^{20}$ = what?

SECTION VI.

BINOMIAL THEOREM.

1) What is the First power of $a + b$?

2) $(a + b)\ 1$ = what?

3) What is the Second power, or square of $a + b$?

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* The pupil ought to recollect that the number belonging to a logarithm which has several negative units at the end, is to be preceded by as many ciphers as the logarithm has negative units. The Decimal point must be made after the first cipher on the left.

4) $(a+b)^2 =$ what?

5) Raise $a+b$ to the 3d power.

6) $(a+b)^3 =$ what?

7) $(a+b)^4 =$ what?
$(a-b)^4 =$ what?

8) $(a+b)^5 =$ what?
$(a-b)^5 =$ what?

9) $(a+b)^6 =$ what?
$(a-b)^6 =$ what?

10) $(a+b)^7 =$ what?
$(a-b)^7 =$ what?

11) $(a+b)^8 =$ what?
$(a-b)^8 =$ what?

12) $(a+b)^9 =$ what?
$(a-b)^9 =$ what?

13) $(a+b)^{10} =$ what?
$(a-b)^{10} =$ what?

14) $(1+x)^3 =$ what?
$(1-x)^3 =$ what?

15) $(1+x)^4 =$ what?
$(1-x)^4 =$ what?

16) $(1+x)^5 =$ what?
$(1-x)^5 =$ what?

17) $(2+3x)^5 =$ what?
$(2-3x)^5 =$ what?

18) $(5+4x)^4 =$ what?
$(5-4x)^4 =$ what?

19) $(\frac{1}{2}x+2y)^4 =$ what?
$(\frac{1}{2}x-2y)^4 =$ what?

20) $(3+2x^2)^5 =$ what?
$(3-2x^2)^5 =$ what?

21) $(3ac+2bd)^2 =$ what?
$(3ac-2bd)^2 =$ what?

22) $(\sqrt{a}+\sqrt{b})^2 =$ what?
$(\sqrt{a}-\sqrt{b})^2 =$ what?

23) $(\sqrt{a}+\sqrt{b})^4 =$ what?
$(\sqrt{a}-\sqrt{b})^4 =$ what?

24) $(a\sqrt{b}+c\sqrt{d})^2 =$ what?
$(a\sqrt{b}-c\sqrt{d})^2 =$ what?

General Formula. $(a+b)^n = a^n + n\,a^{n-1}\,b +$

$$\frac{n\,(n-1)}{1\times 2}\,a^{n-2}\,b^2 + \frac{n\,(n-1)\,(n-2)}{1\times 2\times 3}\,a^{n-3}\,b^3 +$$

$$\frac{n\,(n-1)\,(n-2)\,(n-3)}{1\times 2\times 3\times 4}\,a^{n-4}\,b^4 +$$

$$\frac{n\,(n-1)\,(n-2)\,(n-3)\,(n-4)}{1\times 2\times 3\times 4\times 5}\,a^{n-5}\,b^5 + \&\text{c. } b\ .$$

APPLICATION OF THE GENERAL FORMULA TO THE EXTRACTION OF IMPERFECT ROOTS.

1) $\sqrt{1+x} = (1+x)^{\frac{1}{2}} =$ what ? *

2) $\sqrt{1-x} = (1-x)^{\frac{1}{2}} =$ what ?

3) $\sqrt[3]{1+x} = (1+x)^{\frac{1}{3}} =$ what ?

4) $\sqrt[3]{1-x} = (1-x)^{\frac{1}{3}} =$ what ?

5) $\sqrt[4]{1+x} = (1+x)^{\frac{1}{4}} =$ what ?

6) $\sqrt[4]{1+x} = (1+x)^{\frac{1}{4}} =$ what ?

7) $\sqrt{a+x} = (a+x)^{\frac{1}{2}} =$ what ?

8) $\sqrt{a-x} = (a-x)^{\frac{1}{2}} =$ what ?

9) $\sqrt{a^2+x^2} = (a^2+x^2)^{\frac{1}{2}} =$ what ?

10) $\sqrt{a^2-x^2} = (a^2-x^2)^{\frac{1}{2}} =$ what ?

11) $\sqrt[3]{a^3+x^3} = (a^3+x^3)^{\frac{1}{3}} =$ what ?

12) $\sqrt[3]{a^3-x^3} = (a^3-x^3)^{\frac{1}{3}} =$ what ?

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* The answer may also be obtained by extracting the square roots in the common way.

SECTION VII.

A. Arithmetical Progressions.

Formula. When a denotes the first term of the progression, d the difference, n the number of terms, l the last term, and s the sum of all the terms ; then we have

$$l = a + (n - 1) \times d ;$$

that is, the last term of an Arithmetical progression is obtained by adding to the first term the difference, taken as many times as there are terms less one ; and

$$s = \frac{(a + l)\,n}{2} ;$$

that is, the sum of all the terms of an Arithmetical progression is obtained by multiplying the sum of the first and last term by the number of terms, and dividing the product by 2.

EPAMPLES.

1) The First term of an Arithmetical progression is 1, the difference 1, and the number of terms 14 ; What is the 14th term, and the sum of all 14 terms ?

2) The first term of an Arithmetical progression is 2
the difference 3
the number of terms 13
What is the 13th term, and what the sum of all 13 terms ?

3) The first term is 3
the difference 2
and the num. of terms 20
What is the twentieth term, and what the sum of all 20 terms ?

4)	The first term is	4	What is the 100th term,
	the difference	1	and what the sum of
	the number of terms	100	all 100 terms?
5)	The first term is	5	What is the 25th term,
	the difference	5	and what the sum of
	the number of terms	25	all 25 terms?
6)	The first term is	7	What is the 10th term,
	the difference	$\frac{1}{2}$	and what the sum of
	the number of terms	10	all 10 terms?
7)	The first term is	7	What is the 16th term,
	the difference	$\frac{1}{4}$	and what the sum of
	the number of terms	16	all 16 terms?
8)	The first term is	$\frac{1}{2}$	What is the 10th term,
	the difference	$\frac{1}{4}$	and what the sum of
	the number of terms	10	all 10 terms?
9)	The first term is	$\frac{1}{3}$	What is the 12th term,
	the difference	$\frac{1}{6}$	and what the sum of
	the number of terms	12	all 12 terms?
10)	The first term is	$2\frac{1}{2}$	What is the 100th term,
	the difference	$\frac{1}{3}$	and what the sum of
	the number of terms	100	all 100 terms?
11)	The first term is	$\frac{3}{4}$	What is the 26th term,
	the difference	$\frac{1}{8}$	and what the sum of
	the number of terms	26	all 26 terms?
12)	The first term is	$\frac{2}{3}$	What is the 16th term,
	the difference	$\frac{1}{3}$	and what the sum of
	the number of terms	16	all 16 terms?
13)	The first term is	0	What is the 14th term,
	the difference	$\frac{1}{2}$	and what the sum of
	the number of terms	14	all 14 terms?
14)	The first term is	$\frac{4}{5}$	What is the 32d term,
	the difference	$\frac{1}{10}$	and what the sum of
	the number of terms	32	all 32 terms?

15) The first term is 1
the difference $\frac{1}{8}$
the number of terms 36
What is the 36th term, and what the sum of all 36 terms?

16) The first term is $\frac{1}{10}$
the difference $\frac{1}{2}$
the number of terms 8
What is the 8th term, and what the sum of all 8 terms?

17) The first term is 10
the difference 10
the number of terms 100
What is the 100th term, and what the sum of all 100 terms?

18) The first term is $\frac{3}{8}$
the difference $\frac{1}{4}$
the number of terms 50
What is the 50th term, and what the sum of all 50 terms?

19) The first term is $\frac{1}{2}$
the difference $\frac{1}{3}$
the number of terms 22
What is the 22d term, and what the sum of all 22 terms?

20) The first term is $\frac{3}{4}$
the difference $\frac{1}{5}$
the number of terms 10
What is the 10th term, and what the sum of all 10 terms?

21) The first term is $\frac{4}{5}$
the difference $\frac{1}{5}$
the number of terms 14
What is the 14th term, and what the sum of all 14 terms?

22) The first term is -1
the difference $+1$
the number of terms 12
What is the 12th term, and what the sum of all 12 terms?

23) The first term is 3
the difference $2\frac{1}{2}$
the number of terms 25
What is the 25th term, and what the sum of all 25 terms?

24) The first term is 4
the difference $\frac{1}{8}$
the number of terms 40
What is the 40th term, and what the sum of all 40 terms?

B. Geometrical Progression.

Formula. When a denotes the first term of a Geometrical Progression, e the quotient between two consecutive terms, l the last term, n the number of terms, and s the sum of all the terms; then we have

$$l = a\,e^{n-1};$$

that is, the last term of a Geometrical Progression is obtained by multiplying the first term by the quotient, raised to the number of terms less 1st power; and

$$s = \frac{e\,l - a}{e - 1},$$

that is, in order to obtain the sum of all the terms of a Geometrical Progression multiply the quotient by the last term, subtract the first term from this product, and divide the remainder by the quotient diminished by unity.

APPLICATIONS.

1)	The first term is	1	What is the 7th term, and what the sum of all 7 terms?
	the Quotient	2	
	the number of terms	7	
2)	The first term is	4	What is the 10th term, and what the sum of all 10 terms?
	the Quotient	3	
	the number of terms	10	
3)	The first term is	5	What is the 9th term, and what the sum of all 9 terms?
	the Quotient	4	
	the number of terms	9	
4)	The first term is	1	What is the 10th term, and what the sum of all 10 terms?
	the Quotient	3	
	the number of terms	10	
5)	The first term is	2	What is the 12th term, and what the sum of all 12 terms?
	the Quotient	4	
	the number of terms	12	

6) The first term is 1 What is the 10th term,
the Quotient $\frac{1}{2}$ and what the sum of
the number of terms 10 all 10 terms ?

7) The first term is 1 What is the 6th term,
the Quotient $\frac{1}{4}$ and what the sum of
the number of terms 6 all 6 terms ?

8) The first term is 2 What is the 8th term,
the Quotient $\frac{1}{2}$ and what the sum of
the number of terms 8 all 8 terms ?

9) The first term is 4 What is the 9th term,
the Quotient $\frac{1}{4}$ and what the sum of
the number of terms 9 all 9 terms ?

10) The first term is 9 What is the 7th term,
the Quotient $\frac{7}{4}$ and what the sum of
the number of terms 7 all 7 terms ?

11) The first term is $6\frac{1}{4}$ What is the 8th term,
the Quotient $\frac{3}{2}$ and what the sum of
the number of terms 8 all 8 terms ?

12) The first term is 6 What is the 6th term,
the Quotient $\frac{3}{4}$ and what the sum of
the number of terms 6 all 6 terms ?

13) The first term is 6 What is the 4th term,
the Quotient $\frac{1}{4}$ and what the sum of
the number of terms 4 all 4 terms ?

14) The first term is 8 What is the 15th term,
the Quotient $\frac{1}{2}$ and what the sum of
the number of terms 15 all 15 terms ?

15) The first term is 8 What is the 12th term,
the Quotient $\frac{1}{4}$ and what the sum of
the number of terms 12 all 12 terms ?

16) The first term is $3\frac{1}{3}$
the Quotient $\frac{3}{5}$
the number of terms 8
What is the 8th term, and what the sum of all 8 terms ?

17) The first term is 2
the Quotient $1\frac{1}{2}$
the number of terms 6
What is the 6th term, and what the sum of all 6 terms?

18) The first term is 3
the Quotient $\frac{1}{3}$
the number of terms 4
What is the 4th term, and what the sum of all 4 terms ?

19) The first term is 3
the Quotient $\frac{2}{3}$
the number of terms 4
What is the 4th term, and what the sum of all 4 terms ?

20) The first term is $\frac{1}{2}$
the Quotient $\frac{1}{2}$
the number of terms 6
What is the 6th term, and what the sum of all 6 terms ?

21) The first term is $\frac{1}{3}$
the Quotient $\frac{1}{3}$
the number of terms 4
What is the 4th term, and what the sum of all 4 terms ?

22) The first term is $\frac{1}{4}$
the Quotient $\frac{1}{4}$
the number of terms 6
What is the 6th term, and what the sum of all 6 terms ?

23) The first term is $\frac{1}{5}$
the Quotient $\frac{1}{5}$
the number of terms 6
What is the 6th term, and what the sum of all 6 terms ?

24) The first term is 6
the Quotient $\frac{1}{6}$
the number of terms 6
What is the 6th term, and what the sum of all 6 terms?

SECTION VIII.

Equations.

a. SIMPLE EQUATIONS.

a. WITH ONE UNKNOWN QUANTITY.

1) If $3d + 2x + x = 36$, what is x equal to?
2) If $3x + x = 24 - 4$, what is x equal to?
3) If $3x + 2x - x = 32$, what is x equal to?
4) If $3x + 4x - x = 38 - 2$, what is x equal to?
5) If $6x - x = 25$, what is x equal to?
6) If $6x + 5 = 29$, what is x equal to?
7) If $5x + 2x + 3 = 24$, what is x equal to?
8) If $4x - 3 = 21$, what is x equal to?
9) If $3x + 2x - 5 = 30$, what is x equal to?
10) If $40x + 7 - 2 = 125$, what is x equal to?
11) If $3x + 7x + 9 - 19 = 90$, what is x equal to?
12) If $4x = 9 + x$, what is x equal to?
13) If $5x = 18 + 2x$, what is x equal to?
14) If $4x + 3x = 12 + 10 + 2 + x$, what is x equal to?
15) If $5x + 4x = 30 + x - 2 + 4$, what is x equal to?
16) If $4x = 25 - x$, what is x equal to?
17) If $7x = 40 - x$, what is x equal to?
18) If $13x = 30 + 25 + 8x$, what is x equal to?
19) If $10x - 5x = 40 - x - 4$, what is x equal to?

20) If $9x + 2x = 36 - 4x + 24$, what is x equal to?

21) If $3x - 4 = x + 6$, what is x equal to?

22) If $5x - 5 = 2x + 19$, what is x equal to?

23) If $8x - 5 = 13 + 2x$, what is x equal to?

24) If $8x - 5 = 13 - x$, what is x equal to?

25) If $14 - x = 18 - 3x$, what is x equal to?

26) If $20 - 3x = 36 - 7x$, what is x equal to?

27) If $8x - 5 = 13 - 7x$, what is x equal to?

28) If $\frac{1}{2}x + \frac{1}{3} = 30x$, what is x equal to?

29) If $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x = 65$, what must x be equal to?

30) If $\frac{3}{4}x + \frac{2}{3}x - \frac{1}{6}x = 60$, what is x equal to?

31) If $\frac{5}{8}x + \frac{2}{3}x - \frac{1}{4}x = 46\frac{2}{3}$, what is x equal to?

32) If $\frac{5}{6}x - 3 = \frac{3}{4}x - 1$, what is x equal to?

33) If $\frac{4}{5}x - 2\frac{1}{2} = \frac{2}{3}x + 1\frac{1}{2}$, what is x equal to?

34) If $13\frac{3}{4} - \frac{x}{2} = 2x - 8\frac{3}{4}$, what is x equal to?

35) If $\frac{2}{3}x + \frac{4}{5}x + \frac{3}{4}x + \frac{5}{6}x = 3\frac{1}{20}$, what is x equal to?

36) If $2x + 7 + \frac{3}{6}x = 2x - 23$, what is x equal to?

37) If $8x - 5 = 13 - 7x$, what must x be equal to?

38) If $\frac{4}{5}x - 1\frac{1}{2} = 14 - \frac{1}{2}x$, what must x be equal to?

39) If $12\frac{1}{4} + 3x - 6 - \frac{7x}{3} = \frac{3x}{4} - 5\frac{3}{8}$ what must x be equal to?

40) If $x - 4x = -12$, what is x equal to?

41) If $\frac{1}{3}x - \frac{3}{4}x = -4\frac{1}{5}$, what is x equal to?

42) If $\frac{3x}{5} - \frac{7x}{10} + \frac{3x}{4} - \frac{7x}{8} = -15$, what is x equal to?

43) If $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 24 + x$, what is x equal to?

44) If $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 156 - 13\,x$, what is x equal to?

45) If $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 7\,x - 712 + \frac{x}{5}$, what is x equal to?

46) If $4.5\,x - 2.75\,x = 4500$, what is x equal to?

47) If $7.5\,x - 3.45 = 3.25\,x + 41.65$, what is x equal to?

48) If $3.25\,x - 5.007 - x = 0.2 - 0.34\,x$, what is x equal to? (Find 3 or 4 decimals of the answer.)

49) If $13.2\,x - \frac{3}{4}\,x + 7.695 = \frac{x}{5} + 7834.5$, what is x equal to?

50) If $\frac{7.53\,x}{18} + 100 = \frac{2\,x}{5} + 3.86 - \frac{x}{6}$, what is x equal to?

51) If $a\,x = b$, what is x equal to?

52) If $a\,x + b\,x = c$, what is x equal to?

53) If $a\,x + c = d$, what is x equal to?

54) If $a\,x + c = b\,x + d$, what is x equal to?

55) If $\frac{1}{x} = 40$, what is x equal to?

56) If $\frac{1}{x} = a$, what is x equal to?

57) If $\frac{2}{x} = 16$, what is x equal to?

58) If $\frac{a}{x} = b$, what is x equal to?

59) If $\frac{3}{x} = a + 1$, what is x equal to?

60) If $\frac{3\,a + 1}{x} + 5 = \frac{6}{x}$, what is x equal to?

61) If $\frac{a}{x} = b + c$, what is x equal to ?

62) If $4\,x - 16 = 0$, what is x equal to ?

63) If $a\,x + b\,x - c = 0$, what is x equal to ?

64) If $a\,x + b\,x = c + d - e\,x$, what is x equal to ?

65) If $\frac{4}{x} + \frac{5}{x} = 3$, what is x equal to ?

66) If $\frac{a}{x} + \frac{b}{x} = c$, what is x equal to ? (Multiply the whole equation by x.)

67) If $\frac{a}{x} + \frac{b}{x} + \frac{1}{x} = c\,d$, what is x equal to ?

68) If $46 - \frac{1}{x} = 20 + \frac{2}{x}$, what is $x =$ to ?

69) If $a + \frac{1}{x} = b + c + \frac{d}{x}$, what is x equal to ?

70) If $\frac{x}{a} - 1 - \frac{d\,x}{c} + 3\,a\,b = 0$, what is x equal to ?

71) $\sqrt{x} = 4$, what is x equal to ?

72) $\sqrt[3]{x} = 5$, what is x equal to ?

73) $\sqrt{x} = a$, what is x equal to ?

74) $\sqrt[3]{x} = a$, what is x equal to ?

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b. EQUATIONS WITH TWO AND MORE UNKNOWN QUANTITIES.

1) If $\left.\begin{array}{r} x + y = 10 \\ \text{and } x - y = 4 \end{array}\right\}$ What are x and y equal to ?

2) If $\left.\begin{array}{r} 2\,x + y = 24 \\ \text{and } 3\,x + y = 34 \end{array}\right\}$ What are x and y equal to ?

3) If $3x + 2y = 33$
and $2x + 4y = 40$
What are x and y equal to?

4) If $5x + 4y = 73$
and $4x + 5y = 73$
What are x and y equal to?

5) If $3x + 2y = 118$
and $x + 5y = 191$
What are x and y equal to?

6) If $5x = 4y$
and $x + y = 9$
What are x and y equal to?

7) If $2x + \frac{1}{2}y = 12$
and $13x - y = 61$
What are x and y equal to?

8) If $\frac{1}{2}x + \frac{1}{3}y = 9$
and $4x + 3y = 65$
What are x and y equal to?

9) If $\frac{1}{2}x + \frac{1}{4}y = 4$
and $\frac{5}{6}x - \frac{1}{2}y = 3$
What are x and y equal to?

10) If $2x - y = 1$
and $\frac{5}{6}x + \frac{2}{3}y = 14$
What are x and y equal to?

11) If $7y = 2x - 3y$
and $19x = 60y + 621\frac{1}{4}$
What are x and y equal to?

12) If $4x - \frac{1}{3}y = 90$
and $\frac{1}{8}x - \frac{1}{6}y = 0$
What are x and y equal to?

13) If $\dfrac{3}{4+y} = \dfrac{4}{7+x}$
and $3x + 4y = 7$
What are x and y equal to?

14) If $x + y = 18.73$
and $0.5x + 7.9y = 54.32$
Find four Decimals of x and y.

15) If $3.5x - 4.7y = 22.45$
and $0.7x - 3.5y = 17.25$
Find three Decimals of x and y.

16) If $x + y = a$
and $x - y = b$
What are x and y equal to?

17) $\left.\begin{array}{r} \text{If } a x + y = b \\ \text{and } d x + y = c \end{array}\right\}$ What are x and y equal to?

18) $\left.\begin{array}{r} \text{If } a x + b y = c \\ \text{and } d x + f y = g \end{array}\right\}$ What are x and y equal to?

19) $\left.\begin{array}{r} \text{If } a x = b y \\ \text{and } x + y = c \end{array}\right\}$ What are x and y equal to?

20) $\left.\begin{array}{r} \text{If } a x = b y \\ \text{and } a x + c y = d \end{array}\right\}$ What are x and y equal to?

21) $\left.\begin{array}{r} \text{If } \dfrac{x}{a} + \dfrac{y}{b} = d \\ \text{and } \dfrac{x}{a} - \dfrac{y}{c} = e \end{array}\right\}$ What are x and y equal to?

22) $\left.\begin{array}{r} \text{If } \dfrac{a}{b + y} = \dfrac{b}{a + x} \\ \text{and } a x + b y = d \end{array}\right\}$ What are x and y equal to?

23) $\left.\begin{array}{r} \text{If } x + y = 10, \\ x + z = 19, \\ \text{and } y + z = 23, \end{array}\right\}$ What must x, y and z be equal to?

24) $\left.\begin{array}{r} \text{If } x - y = 1, \\ x - z = 2, \\ \text{and } y - z = 1, \end{array}\right\}$ What must x, y and z be equal to?

25) $\left.\begin{array}{r} \text{If } 3 x + 5 y = 161, \\ 7 x + 2 z = 209, \\ \text{and } 2 y + z = 89, \end{array}\right\}$ What must x, y and z be equal to?

26) $\left.\begin{array}{r} \text{If } y + \frac{1}{4} x = 41, \\ x + \frac{1}{4} z = 20\frac{1}{2}, \\ \text{and } y + \frac{1}{5} z = 34, \end{array}\right\}$ What must x, y and z be equal to?

27) $\left.\begin{array}{r} \text{If } \frac{1}{2} y - \frac{1}{4} x = 3, \\ \frac{1}{3} y - \frac{1}{3} z = 1\frac{1}{3}, \\ \text{and } \frac{1}{6} x - \frac{1}{5} z = \frac{2}{5}, \end{array}\right\}$ What must x, y and z be equal to?

28) If $x + y = a$,[*]
$x + z = b$,
and $y + z = c$,
What must x, y and z be equal to?

29) If $a x + b y = c$,
$d x + e y = f$,
and $g y + h z = e$,
What must x, y and z be equal to?

30) If $x + y + z = 12$,
$x + y - z = 2$,
and $x - y + z = 4$,
What must x, y and z be equal to?

31) If $x + y + z = 24$,
$5 y = 4 x$,
and $5 z = 3 x$,
What must x, y and z be equal to?

32) If $4 x + 7 y = 147$,
$3\frac{1}{3} x - \frac{1}{6} z = 77$,
and $2 x + y + 9 z = 227$,
What are x, y and z equal to?

33) If $2 x + 5 y - 7 z = -288$,
$5 x - y + 3 z = 227$,
and $7 x + 6 y + z = 297$,
What are x, y and z equal to?

34) If $x + y + z = 30$,
$8 x + 4 y + 2 z = 50$,
and $27 x + 9 y + 3 z = 64$,
What are x, y and z equal to?

35) If $18 x - 7 y - 5 z = 11$,
$-\frac{2}{3} x + 4\frac{2}{3} y + z = 108$,
and $\frac{3}{4} x + \frac{1}{2} y + 3\frac{1}{2} z = 80$,
What are x, y and z equal to?

36) If $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{7}{12}$,
$\dfrac{1}{x} + \dfrac{1}{z} = \dfrac{3}{4}$,
and $\dfrac{1}{y} + \dfrac{1}{z} = \dfrac{5}{6}$,
What are x, y and z equal to?

(Subtract immediately the third Equation from the second, and add the result to the third equation, by which means you obtain an equation in z only.)

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[*] This and the following problem may be omitted by very young pupils.

37) If $\left.\begin{array}{l} \dfrac{1}{x} + \dfrac{1}{y} = a, \\ \dfrac{1}{x} + \dfrac{1}{y} = b, \\ \text{and } \dfrac{1}{y} + \dfrac{1}{z} = c, \end{array}\right\}$ What are x, y, z, equal to?

38) If $\left.\begin{array}{rl} & \frac{1}{4}x + \frac{1}{3}y + \frac{1}{2}z + u = 4, \\ & x + y + z + u = 10, \\ & 4x + 3y + 2z + u = 30, \\ \text{and} & 7x + 5y + 3z + u = 50, \end{array}\right\}$ What are x, y, z, u, equal to?

(Subtract the first Equation successively from the 2d, 3d, and 4th Equations, to get rid of the u.)

39) If $\left.\begin{array}{rl} & x + y + z + u = 1, \\ & 16x + 8y + 4z + 2u = 9, \\ & 81x + 27y + 9z + 3u = 36, \\ \text{and} & 256x + 64y + 16z + 4u = 100, \end{array}\right\}$ What must x, y, z, u, be equal to?

(Multiply the first Equation in order by 2, 3, 4, and subtract the products respectively from the 2d, 3d, and 4th Equation to expel the u.)

40) If $\left.\begin{array}{rl} & 4x + 3y = 43, \\ & 2x + z = 17, \\ & x + 2u = 9, \\ \text{and} & 3u + z = 6, \end{array}\right\}$ What must x, y, z, u, be equal to?

B. *Quadratic Equations.*

Formula. If $x^2 + ax = b$

then $x = -\dfrac{a}{2} \pm \sqrt{b + \frac{1}{4}a^2}$

1) If $4x^2 = 144$, what is x equal to?

2) If $\dfrac{x^2}{4} = 20\frac{1}{4}$, what is x equal to?

3) If $2\frac{1}{2} x^2 = 40$, what is x equal to ?

4) If $x^2 + 6 x = 27$, what are the two values of x ?

5) If $x^2 - 6 x = 72$, what are the two values of x ?

6) If $x^2 - 14 x = - 45$, what are the two values of x ?

7) If $x^2 - 7 x = - 3\frac{1}{4}$, what are the two values of x ?

8) If $x^2 - 5\frac{3}{4} x = 18$, what are the two values of x ?

9) If $x^2 + 9\frac{1}{2} x = 54$, what are the two values of x ?

10) If $x^2 + \frac{3}{4} x = 70$, what are the two values of x ?

11) If $x^2 + 2\frac{3}{4} x = 73\frac{1}{2}$, what are the two values of x ?

12) If $x^2 - \frac{1}{2} x = \frac{3}{16}$, what are the two values of x ?

13) If $x^2 - \frac{1}{3} x = 0$, what are the two values of x ?

14) If $x^2 + \frac{117}{50} = 6\frac{1}{4} x$, what are the two values of x ? (Bring $6\frac{1}{4} x$ on the left side, and $\frac{117}{50}$ on the right side of the equation.)

15) If $x^2 - \frac{1}{6} = \frac{1}{6} x$, what are the two values of x ?

16) If $40 - 3 x = x^2$, what are the two values of x ?

17) If $3 x^2 - 2 x = 65$, what are the two values of x ? (Divide first the whole question by 3.)

18) If $15 x^2 - 622 x = - 638$, what are the two values of x ? (Divide first the whole equation by 15.)

19) If $\frac{7}{2} x^2 - \frac{47}{4} x = 41\frac{1}{4}$, what are the two values of x ? (Divide first the whole equation by $\frac{7}{2}$.)

20) If $9\frac{1}{3} x^2 - 90\frac{1}{3} x = - 195$, what are the two values of x ? (Divide first the whole equation by $9\frac{1}{3}$.)

21) If $2 x^2 - 120 = 14 x$, what are the two values of x ?

22) If $252 = 3 x^2 + x$, what are the two values of x ?

23) If $- 12 + 16 x = 4 x^2$, what are the two values of x ?

24) If $-1\frac{7}{8}+5\frac{1}{2}x=4x^2$, what are the two values of x?

25) If $196+7x=5x^2$, what are the two values of x?

26) If $\frac{7}{8}-6x=8x^2$, what are the two values of x?

27) If $4x-3x^2=\frac{17}{25}$, what are the two values of x?

28) If $96-4x=2x^2$, what are the two values of x?

29) If $3x^2+x=7$, find the two approximate values of x as far as three decimals.

30) If $2x^2-x=5$, find the two approximate values of x as far as four decimals.

31) If $3x^2+x=7$, find the approximate values of x as far as three decimals.

32) If $4x^2-5x=14$, find the approximate values of x to four decimals.

33) If $3x^2-6x=-30$, are the two values of x real or imaginary?

34) If $2x^2-4x=-25$, are the two values of x real or imaginary?

35) If $\frac{x}{x+60}=\frac{7}{3x+51}$, what are the two values of x?

36) If $\frac{40}{x-5}+\frac{27}{x}=13$, what are the two values of x?

37) If $\frac{8x}{x+2}-6=\frac{20}{3x}$, what are the two values of x?

38) If $\frac{48}{x+3}=\frac{165}{x+16}-5$, what are the values of x?

39) If $adx-acx^2=bcx-bd$, what are the two values of x?

40) If $\frac{a^2x^2}{f^2}-\frac{2ax}{5}=-\frac{f^2}{g^2}$, what are the two values of x?

SECTION IX.

PROBLEMS FOR SIMPLE EQUATIONS WITH ONE UNKNOWN QUANTITY.

A. Comparison of the Unknown Quantity with one or more known ones.

a PROBLEMS IN WHICH THE UNKNOWN QUANTITY IS DETERMINED BY A MULTIPLE.

[The problems belonging to this section are so arranged, that the pupils may also solve them in their heads by a simple process of reasoning.]

1) If I had $100 more than I have, I should be able to pay all my debts? How much money do I need?

2) Three times as much money as I have would amount to $24. How much money have I?

3) Two gentlemen arrived late in the evening at an Inn. To obtain a room, one offered to pay double, and the other 3 times the ordinary price. Both together paid $5. What was the ordinary price of a room?

4) If I had twice as much money as I have, and again five times as much, I should have $14. How much money have I?

5) If my father would give me 3 times as much money as I have, and my mother 4 times as much, I should have $24. How much money have I?

6) Two gentlemen saw a sum of money lying on the table. The first said, " I have 3 times as much money as

there is on that table." "Well," replied the other, "I have ten times as much;" both together had $65. How much money was there on the table?

7) A merchant employed a certain capital in trade, and by prudent management cleared every year a sum equal to that capital. At the end of the 3d year his capital amounted to $8,000. What was his first capital?

8) I had once $40, from which I spent a certain sum; but retained 3 times as much as I spent. How much did I spend? And how much did I retain?

9) A and B were talking of a certain sum of money: A said, "I have 3 times as much;" upon which B replied; "Well, then, I have exactly 4 times as much as you;" both together had $750. What sum were they talking of?

10) A complained he owed as much again as he was worth; B avowed he owed 3 times as much as A; whereupon C remarked that his debt was 5 times as large as B's. All three together owed $30,000. How much money was A worth, and how much did each one owe?

11) A, B, C, were talking of their ages. A said, I am twice as old as B; B said, I am twice as old as C; together they were 160 years old. What is C's age? And what are the ages of A and B?

12) Two, Three, Four, and Five times my money, together with the money I have, amount to $45,000. How much money have I?

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b. PROBLEMS IN WHICH THE UNKNOWN QUANTITY IS DETERMINED BY ITS PARTS.

1) Half of a certain number is 50. What is the num-

ber? What would be the number if $\frac{1}{5}$ of it amounted to 50?

2) Two fifths of a certain number amount to 10. What is the number? Nine thousand dollars are three fourths of my fortune. What is my whole fortune?

3) If one half and one fourth be taken from a certain number, 15 remain, what is the number?

4) A Draper had a piece of cloth from which he sold the fourth and third part, and had still 20 yards left. How many yards did the whole piece contain?

5) How much money have I in my pocket, if the fourth and fifth part of it amount together to 9 dollars.

6) A gamester lost in the first game $\frac{1}{3}$, in the 2d $\frac{1}{4}$, and in the 3d $\frac{1}{5}$ of all the money he had with him. Upon counting he finds he has but $13 left. How much money had he at first?

7) Two gentlemen wished to buy a horse. Upon counting their money they found that one had but $\frac{1}{7}$ and the other only $\frac{1}{8}$ of the whole money that was asked for him, and yet they had together $32. What was the price of the horse?

8) A servant was to receive $30 a year, and a suit of clothes; but his master dismissing him at the end of 4 months ($\frac{1}{3}$ year) gave him nothing but a suit of clothes. What must have been the value of this suit to be a proper compensation for four months' service? *

9) I put a certain sum of money in my pocket; half of it I paid for sugar, $\frac{1}{3}$ for coffee, $\frac{1}{12}$ for rice, $\frac{1}{6}$ for tea, and the remaining $$7\frac{1}{2}$ I spent for cigars. How much money did I put in my pocket?

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* One third of the value of the suit, added to one third of 30 dollars must be the value of the whole suit.

10) An officer giving an account of an engagement he had with the enemy, reported that half the men he commanded were taken prisoners, $\frac{1}{4}$ were killed, $\frac{1}{7}$ severely wounded; and that in consequence of this he had but 3 men left for service. How many men had he before the engagement ?

11) A gentleman was asked how old he was, upon which he answered, $\frac{1}{5}$ of my age I passed in childhood, $\frac{7}{8}$ in my youth, $\frac{1}{2}$ in my manhood, and it is now 14 years that I call myself an old man. How old was he ?

12) Were I to spend $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of the money I have, I should want $8. How much money have I ?

13) A watchman being asked the hour of the night, answered, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of it is just one hour more than the present time. What o'clock was it ?

14) A gamester lost in the first game $\frac{1}{6}$, and in the 2d $\frac{1}{10}$ of the money he had with him ; but in the 3d game won $\frac{1}{3}$ of it back. Upon counting his money, he found he had won $3. How much money had he at first ?

15) A gambler bet with three different persons ; with the first $\frac{1}{3}$, with the 2d $\frac{1}{4}$, and with the 3d $\frac{1}{8}$ of his whole fortune. After having lost the two first bets and won the third he finds himself $11,000 poorer than at first. What was his fortune ?

16) I had once a certain sum of money, of which I spent $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{6}$; but received again $\frac{2}{3}$ of it. Upon counting I had $3 more than at first. How much money had I at first ?

17) A merchant being asked how many hogsheads of wine he had in his store, answered ; last Monday I sold $\frac{1}{2}$ of them, last Tuesday half of the remainder, and last

Wednesday $\frac{1}{2}$ of what was left; but having today received a fresh supply of $\frac{2}{3}$ as many hogsheads as I had at first, I have at this moment but 105 hhds. less than I had at the beginning. How many hogsheads had he at first?

18) A father divided a number of apples among his three children. To the first and oldest he gave $\frac{1}{3}$ of all he had; to the second he gave $\frac{1}{3}$ of the remainder, and to the third $\frac{1}{3}$ of what he had still left. By dividing them in this manner he had but 16 left to himself. How many had he at first?

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c. PROBLEMS IN WHICH THE UNKNOWN QUANTITY IS DETERMINED BY BOTH ITS MULTIPLES AND PARTS.

1) A and B have to pay a bill. A can only pay $\frac{1}{3}$ of it, B has twice as much money as the amount of the bill. When the bill was paid they had together $40 left. What was the amount of the bill, and how much money had each?

2) A gentleman having received 7 times as much money as he already had, was still obliged to lay out $\frac{1}{5}$ of the whole to pay his debts; after which he had but $128 left. How much money had he at first?

3) I multiply a certain number by 4 and divided the product by 3, upon which I obtain 32. What is the number?

4) A gentleman put out his capital at 5 per cent interest, and at the end of 1 year received back $2,100 capital and interest. What was his capital?

[Remark. 5 per cent $= \frac{1}{20}$ of the whole capital, consequently he receives back $\frac{21}{20}$ of his capital.]

5) What is the amount of a capital which at 5 per cent

simple interest, yields in three years $1725 capital and interest? (He will receive $\frac{23}{20}$ of his capital, or $\frac{23}{20} x$.)

6) What is the amount of a capital which at 4 per cent simple interest, yields in 4 years $2,000 capital and interest?

7) What is the amount of a capital which with 6 per cent simple interest, increases in 5 years to $1,300?

8) Three shepherds, A, B, and C, upon counting their sheep, find that B, has 3 times as many as A; but that C has only $\frac{1}{4}$ as many as B. They have in all 19 sheep. How many had each?

9) A merchant finds, by a fortunate speculation with his capital he has gained 15 per cent, and that by this means it has increased to $15,571. What was his capital?

10) The rent of an estate is this year by 8 per cent greater than it was the last year. This year's rent is $1,890. What was the rent of last year?

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d. PROBLEMS IN WHICH THE UNKNOWN QUANTITY IS DETERMINED BY COMPARISON TO ITSELF.

1) Ten times as many dollars as I have, amount to the same as 10 dollars added to my money. How many dollars have I?

2) What number multiplied by 19 gives the same product as 6 added to it?

3) A has twice as much money as B; moreover it is known that A has $12 more than B. How much money has B? And how much had A?

4) If I had 3 times as much money as I have, I should have $18 more than I now have. How much money have I?

5) A gentleman puts out his money at two different places. At one place he puts $\frac{1}{3}$, and at the other $\frac{1}{4}$. It is known that at the first place he has $1,000 more than at the second. What is the whole amount of his money?

6) Since $\frac{1}{3}$ of my money is employed in trade, I can only invest $\frac{1}{5}$ of it in bank stock; because I wish to have $2,000 more in trade than in bank stock. What is the whole amount of my money?

7) $\frac{4}{5}$ of my money exceeds $\frac{1}{2}$ of it by $4,500. What is the amount of money?

8) The difference between $\frac{1}{2}$ and $\frac{1}{3}$ of a man's fortune is $6,000. What is his fortune?

9) A gentleman is 20 years older than his son; his son is $\frac{2}{3}$ as old as his father. How old is each?

10) B's fortune is only $\frac{3}{4}$ of A; the difference between their fortunes is $1,200. How much has each?

11) From a basket of apples I gave a number of children 4 apiece; but finding that in this way I had 44 apples left, I took them all back and gave them 6 apples apiece, upon which I had but 12 left. How many children were there, and how many apples had I at first?

12) A mechanic had a number of workmen, among whom he wished to distribute a certain sum of money. Giving them 9 pence apiece he would have 2*s*. 8*d*. sterling left, but if he wished to give them 11 pence apiece, he would lack 2*s*. 9*d*. How many workmen had he; and what was the sum of money he wished to divide among them?

13) A person wishing to buy some sugar, found that if he gave but 7*d*. per lb. he would have 10*d*. left, but if he gave 8*d*. per lb. he would lack 2*s*. 6*d*. How many lbs. of sugar did he buy?

14) A gentleman came into a store and demanded of a certain commodity. The merchant told him, that at that moment he had not the precise article the gentleman called for; but two other sorts, of which the one was only $\frac{3}{4}$ and the other $\frac{7}{8}$ of the price the gentleman named; and that these sorts differed from each other $10 per cwt. What is the price per cwt. of the article the gentleman called for; and what is the price per cwt. of each sort the merchant had in his store?

15) A person wishing to buy a house, draws a certain sum from each of his debtors. If each pays him $250 he still lacks $2,000; but if each pays him $340 he has $880 more than he needs. What is the price of the house? How many debtors has he? And how much must he exact from each?

16) A person having received $20, went into a store to buy himself some broadcloth for a cloak. The store keeper showed him two different kinds. If he bought of the first, which was $3 a yard, he would have as many dollars left as he would lack if he had wished to buy of the 2d, which was $5 a yard. How many yards did he wish to buy?

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e. PROMISCUOUS PROBLEMS BELONGING TO *a*, *b*, *c*, WITH SEVERAL ADDITIONAL CONDITIONS.

1) After a certain part of a commodity which weighed 40 lbs. was sold; there remained 8 lbs. more than were sold. How many pounds were sold?

2) I once had $42, of which I gave away a certain sum, and there remained still 3 times as much as I had given away. How much did I give away?

3) A schoolmaster was asked how many boys he had in his school. He answered that he had not yet 50; for if he had as many again, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ as many as he actually had, and yet 12 more, he would have but 49. How many scholars had he?

4) A young man being asked his age, answered, that if $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ of his age, and 9 years more were added to his age, he would be 100 years old. How old was he?

5) A farmer's crop surpassed by 7 bushels the amount he had sown. After selling half of his whole crop, he had 306 bushels left. How many bushels had he sown?

6) A traveller inquired how far he had to go to the next town; but as he addressed himself to a young Algebraist, received the following answer. If you add $\frac{3}{4}$ and $\frac{5}{8}$ of the number of miles you have yet to walk to that number itself, and yet $\frac{1}{4}$ mile more, the answer will be 5 miles. How far was he from the next town?

7) How much money did you win? asked one gambler of another. Very little indeed, answered he; for I ought to have won 3 times, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$ as much, and \$1 more, in order to be able to pay the \$100 which I owe you. How much did he win?

8) If I had $3\frac{1}{2}$ as much money as I actually have, I should have \$375 more than I possess. How much money have I?

9) You are very saving, said one journeyman to another. You must by this time have laid up \$400. O, no! answered the 2d, I am very far from it, for I need $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{9}{10}$ as much as I actually have, and yet \$13, to have that sum, (\$400.) How much money had he laid up?

10) Three persons, A, B, C, take a journey. A and B spent together $70 ; A and C $80 ; but B and C $90. What did each one spend ?

11) If I had $25 more than I have, I should have as much more than $50 as I have now less than that sum. How much money have I ?

12) A countryman carrying eggs to market was asked how many there were. If I had as many again as I have, and yet $\frac{1}{2}$ and $\frac{2}{3}$ as many, I should have as many above 50 as I have now less than 50. How many eggs had he ?

B. Dividing a known quantity into one or more unknown parts.

a. PROBLEMS IN WHICH THE PARTS ARE IMMEDIATELY DEDUCED FROM THE WHOLE QUANTITY.

1) A sum of $2,500 is to be divided between two brothers in such a manner that the one is to receive as often $4 as the other receives $1. How much does each receive ?

2) A legacy of $150 is to be divided between two persons in such a way that one is to receive as many times $9 as the other $1. How many dollars does each receive ?

3) $24 are to be divided among 3 persons, A, B, C, so that A receives as many times $2 as B receives $3, and C $4. What is the share of each ?

4) Two gentlemen living 340 miles apart, are setting out on the same day to meet each other. One is travelling at the rate of 8 miles an hour, and the other at the rate of 9 miles. In how many days (or hours) will they meet ?

5) Two persons enter into partnership. One puts in $5,000, and the other $9,000 ; they gain $700. In what manner are they to share this gain ?

6) A gentleman dies and bequeaths to his servants a sum of $3,000 which they are to share according to their wages. The steward had $100 a year, the valet de chambre had $80, the cook $40, and each of the four lackeys had $20 a year. What is the share of each ?

7) A gentleman hires a number of laborers. To one half of them he paid 1*s.* 3*d.* apiece ; but to the other half only 1*s.* apiece ; to the whole he paid 1 guinea, (21*s.*) How many laborers did he hire ?

8) An equal number of men, women, and boys are engaged in a work. Each man receives $1 per day, each woman 50 cents, and each boy 25 cents. Their daily wages amount to $15,75. How many were there of each kind ?

9) A mason, 12 journeymen, and 4 assistants, receive together $72 wages for a certain time. The mason receives $1 daily, each journeyman $\frac{1}{2}$ dollar, and each assistant $\frac{1}{4}$ dollar. How many days must they have worked for this money ?

10) A, B, C, are to build 72 rods of fence. A can build 3 rods a day, B can build 4 rods, and C 5. How many days will they need to finish the whole fence ?

11) Two masons, A and B, are to build a wall of 3,800 cubic feet. A, builds 30 cubic feet daily, B finishes 100 cubic feet in 3 days. If they continue to work in this manner until the whole wall is finished, for how many cubic feet must each mason be paid ?

12) The garrison of a town consists of an equal num-

ber of infantry and cavalry. Each horse soldier receives $5 a month, and each foot soldier $3. The whole monthly pay of the garrison amounts to $9,920. What is the whole number of soldiers in the garrison? And how many are there of each kind?

b. PROBLEMS IN WHICH THE PARTS OF THE UNKNOWN QUANTITY ARE DEPENDING UPON ONE ANOTHER.

1) Two capitalists calculate their fortunes, and it appears that one is twice as rich as the other, and that together they possess $5,400. What is the capital of each?

2) The sum of $2,500 is to be divided between two brothers so that one receives 4 times as many dollars as the other. How much does each receive?

3) A person has $2,640, among which are 4½ times as many notes as specie. How much has he of each sort?

4) Divide the number 837 into two such parts, that the one may be contained in the other 4¼ times. What are these parts?

5) A merchant freighted a vessel of 260 tons, with sugar, and coffee, and tea. He put twice as much coffee on board as tea, and 3½ times as much sugar as coffee. How many tons did she carry of each commodity?

6) In a company of 266 persons, consisting of officers, merchants, and students, there were 4 times as many merchants and twice as many officers as students. How many were there of each class?

7) A fortress has a garrison of 2,600 men; among whom there are 9 times as many foot soldiers, and 3 times

as many artillery soldiers as cavalry. How many are there of each corps?

8) All my journeyings taken together, says a traveller, amount to 3,040 miles; of which I have travelled $3\frac{1}{2}$ times as much by water as on horseback, and $2\frac{1}{2}$ times as much on foot as by water. How many miles did this man travel by each of these three ways?

9) Divide the number 237 into two such parts that the one shall be contained in the other $4\frac{1}{4}$ times. Which are the two parts?

10) Four persons, A, B, C, D, are to share $100 in such a manner that C shall receive twice as much as D; B twice as much as C; and A twice as much as B. What is the share of A? And what are the shares of B, C, D?

11) $280 are to be divided among three brothers in such a way, that each elder brother shall receive double what his next younger brother receives. How much does each receive?

12) Four persons, A, B, C, and D, are to share $100 in such a manner, that B is to receive double of A's share, C double of B's share, and D double of C's share. What is A's share? And what are the shares of B, C, and D?

13) Four soldiers, A, B, C, D, shared a prize of 165 guineas. B received twice as much as A; C three times as much as B; and D 4 times as much as C. How much did each receive?

14) A father dies and leaves a clear fortune of $4,800, which his four sons, A, B, C, and D, are to share in the following manner; A (who is the youngest) is to receive as much as his three brothers, B, C, and D, taken together; B, (who is the next oldest) is to receive as much as C and

D taken together, but C and D are each to receive an equal sum. What is the share of each?

15) Divide $1,177 among A, B, C, in the following manner: B shall receive only $\frac{1}{2}$ of what A receives, and C only $\frac{1}{3}$ of the share of B. What is the share of each?

16) Four persons, A, B, C, and D, are to share $4,200 in such a manner that B shall receive only $\frac{1}{2}$ of A's share, C only $\frac{1}{2}$ of B's, and D only $\frac{1}{2}$ of C's share. What is the share of each?

(What has this problem in common with problem 12?)

17) A father leaves to his three sons, A, B, C, a fortune of $4,570, which according to his will they are to share in the following manner: B is to receive double of A's share; but C only $\frac{1}{3}$ of B's. What is the share of A? And what are the shares of B and C.

18) Three traders, A, B, and C, are to share $360 profits. According to the stock each put in trade. Now it is known that A has put in as much as B and C together; and that B has put in just twice as much as C. What is each one's share of the profit?

19) Four poor persons, A, B, C, D, are to share $560 in proportion to their ages. It is known that D is as old as A, B, and C, taken together, B as old as C and D, and that C and D are of the same age. What is the share of each?

20) $2,400 are to be divided among 5 persons, A, B, C, D, and E, in the following manner: E is to receive as much as A, B, C, and D taken together; D is to receive as much as A, B, and C taken together; C is to receive as much as B and C taken together; and B exactly twice as much as A. What is the share of A? And what are the shares of B, C, D, and E?

c. PROBLEMS BELONGING UNDER THE HEAD OF *a* AND *b*, BUT WITH ONE OR MORE ADDITIONAL CONDITIONS.

1) Two friends, A and B, possessed together $99; A has $1 more than B. How many dollars had each?

2) Two gentlemen, A and B, had together a fortune of $54,000; but A has $5,000 less than B. How many dollars had each?

3) At an election two candidates presented themselves for office. The one that was elected had a majority of 35 votes. The number of votes cast amounted to 250. How many votes had each?

4) Two traders, A and B, made a joint stock of $100. A put in 8½ more than B. How many dollars did each of them put in?

5) Of two numbers one is by 16 greater than the other. Their sum is 96. What are the two numbers?

6) 100 dollars are to be divided between two poor families; the most needful is to receive $4 more than double of the other. How much is each to receive?

7) Two brothers are to share an inheritance of $1,000. Since the oldest brother is already able to provide for himself, the younger is to receive $40 more than three times the share of the other. What is the share of each?

8) Two traders, after leaving off business, are to divide their profits of $1,200 in such a manner, that the one is only to receive $50 more than half the other's share. How much does each receive?

9) Father and son are together 100 years old. If the son were 15 years older than he actually is, then he would

be exactly $\frac{2}{3}$ as old as the father. What is the age of each?

10) Two gentlemen shared an inheritance in such a manner, that the one lacked but $80 in order to have received double of what the other did. What was the share of each?

11) Two butchers bought an ox weighing 360 lbs. They agree to divide him in such a way that the share of the one shall be $\frac{4}{5}$ of the other, lacking 90 lbs. How many lbs. did each receive?

12) 100 lbs. of coffee are to be divided among three families in such a manner that the 2d shall receive 4 lbs. and the 3d 5 lbs. more than the first. How many lbs. does each family receive?

13) $100 are to be divided among A, B, and C, in such a manner that A shall receive $18 more than B; but C $18 less than B. How much does each receive?

14) A father leaves three sons and a clear fortune of $1,600. According to his will, the oldest of them is to receive $100 more than the 2d, and the second $200 more than the youngest. What is the share of each?

15) A widow, according to the will of her deceased husband, shall divide $7,500 with her two sons and three daughters, so that each son shall receive twice as much as each daughter; she herself however is to receive $50 more than all her children taken together. What was her share? And how much did each son and each daughter receive?

16) A company of 90 persons consists of men, women, and children. The men are 4 in number more than the women; the children 10 more than the adults. How

many men, women, and children are there in the company?

17) A forest of 8,000 square feet is to be divided among three farmers, A, B, C, that B may receive 276 square feet less than A; but C 1,112 square feet more than B. How many square feet will each receive?

18) A father gives to his five sons $1,000 which they are to divide according to their ages, so that each elder son shall receive $20 more than his next younger brother. What is the share of the youngest?

19) A certain sum of money is to be divided among three persons, A, B, C, as follows: A shall receive $3,000 more than one half of it, B $1,000 less than the third part, and C is to receive $800 *more* than the 4th part of the whole sum. What is the sum to be divided? And what is each one's share?

20) A dying man bequeaths in his will half of his property to his wife, one sixth to each of his two sons, the 12th part to his servant, and the remaining $600 to the poor. What was the amount of the property bequeathed?

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d. PROBLEMS CONTAINING THE PROGRESSIVE DIVISION OF A KNOWN QUANTITY.

1) Divide $38 into 4 parts, and in such a manner that each following part shall be by $1 greater than the next preceding. What are the four parts?

2) Divide $36 into six parts, and in such a manner that each preceding part shall be by $1 smaller than the next following. What is the first part?

3) A father gives to his five children $1,000, which

they are to share according to their ages, and in such a manner that each elder brother is to receive $20 more than his next younger one. What is the share of each?

4) A gentleman, fond of cider, receives a present of 14 bottles, on condition that he should finish them in a fortnight, and that on each following day he would drink $\frac{1}{8}$ of a bottle more than on the preceding day. How much must he drink the first day? And how much on each following day?

5) A debt of $300 is to be paid in three different instalments, so that the amount of each following instalment shall be only one half of the preceding one. What is the amount of the first instalment?

6) A spendthrift wishes to treat his friends a whole week (seven days) in succession, and in such a manner that on each following day the entertainment is to cost 3 times the amount of the preceding, and yet he does not wish to spend more than $2,186 for that purpose. What must his first entertainment cost him?

7) A merchant by a fortunate speculation doubled annually his floating capital, and found that at the end of the 5th year it had increased to $96,000. What was the amount of his capital at the beginning of the first year?

8) A trader increased his stock annually by $50, and found that at the end of the 10th year he had just doubled it. What was the amount of his stock at the beginning of the first year?

9) A merchant managed his business so well, that at the end of the first year he had increased his stock by $\frac{1}{2}$ its amount. At the end of the 2d year he had increased this accumulated stock again by $\frac{1}{3}$ its amount; and at the

end of the 3d year this new stock again by $\frac{1}{4}$ its amount. He had then just \$10,000. What was his stock at the beginning of the first year?

10) By how many dollars must I increase my capital of \$1,200 annually, in order to double it in 5 years?

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e. DIVISION OF A GIVEN QUANTITY IN A GEOMETRICAL RATIO.

1) A sum of \$2,400 is to be divided between two persons, A and B, in such a manner that the share of A shall be to that of B as 3 to 5. What is the share of each?

2) Divide \$420 in the ratio of 3 to 5. What are the two parts?

3) The sum of \$1,200 is to be divided between two persons, A and B, in such a manner that A's share is to B's as 2 to 7. How much does each receive?

4) A field of 864 square roods is to be divided among three farmers, A, B, C, so that A's part shall be to B's as 5 to 11, and C may receive as much as A and B together. How many square roods does each receive?

5) On an approaching war, three towns, A, B, and C, are to furnish their complements of 594 men. Now the population of A is to that of B as 3 is to 5; while the population of B is to that of C as 8 to 7. How many men must each town furnish?

6) Divide \$1,170 among three persons, A, B, and C, in proportion to their ages. Now B is $\frac{1}{3}$ older than A, and C twice as old as A. How much does each receive?

7) A bankrupt leaves \$21,000 to be divided among four

creditors, A, B, C, and D, in proportion to their claims. Now A's claim is to B's as 2 to 3; B's claim is to C's as 4 to 5; and C's claim to D's as 6 to 7. How much then does each creditor receive?

C. Comparison and Determination of numbers by Addition and Subtraction.

a. PROBLEMS IN WHICH BY CONTINUED ADDITION OR SUBTRACTION ONE QUANTITY IS MADE EQUAL TO ANOTHER.

1) Two persons, A and B, state their fortunes. A says I have $100. B says I have $200. How many times would it be necessary to add $1 to A's fortune to make it equal to B's?

2) Says A to B, I have $40 more than you. How many times $5 must B receive to have as much as A?

3) Two laborers, A and B, have been working together a number of days. A, who has already earned $100, is now dismissed; and B, who has earned but $70, is to remain till he has earned the same sum, ($100.) B receives daily $\frac{2}{3}$ of a dollar. How many days longer is he to work?

4) A man promises to walk 50 miles on condition that he was to walk but $\frac{3}{4}$ of a mile in an hour. How many hours would he need?

(Here the difference from 0 to 50 is to be made up.)

5) Two gamesters, A and B, began to play with different sums of money. A began with $20, B had but $\frac{1}{4}$ of that sum. A by continued losing brought his money down to what B had when they began to play. They played for 50 cents a game. How many games have they played?

6) How many times must I add ⅔ to ⅓ of 20 to make the sum equal to 20 ?

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b. PROBLEMS IN WHICH TWO QUANTITIES ARE MADE EQUAL TO ONE ANOTHER BY ADDING OR SUBTRACTING FROM BOTH.

1) Two farmers, A and B, were ploughing their fields. A had already finished 30 acres, B only 20. Now if B can plough 3 acres a day, and A only 2 acres, how many days must they work before they shall have ploughed the same number of acres ?

2) A courier who had started from a certain place 10 hours ago, is pursued by another from the same place, and by the same way. The first goes 4 miles an hour, and the other 9. How many hours will the second need to overtake the first ?

3) At 1 o'clock the small hand of a clock points at 1, and the large hand at 12. The large hand goes twelve times faster than the small one, (passing over the space of 60 minutes when the small hand passes over 5 minute partitions.) In how many minutes will the large hand overtake the small hand ?

4) A person puts out a capital of $1,000 at 4 per cent interest, and 5 years later, another capital of $1,800, at 5 per cent. If he leaves these two capitals constantly at interest, in how many years will he have drawn the same interest from both ?

5) A person puts out a capital of $5,500 at 4 per cent interest, and 4½ years later another capital of $8,000 at 5 per cent. If he leaves these two capitals constantly at

interest, in how many years will he have drawn the same interest from both?

6) I have two pocket books, A and B. A contains none but two dollar bills; the other, B, contains nothing but three dollar bills. There is the same number of bills in both; but to make their contents equal I must add $500 to A, and $300 to B. How many bills are there in each pocket book? And what amount of money?

7) A father has two sons, one of whom is 20, and the other 25 years of age. He himself is 60 years old. When will his two sons together be as old as he is?

8) Two families, A and B, bought together 110 lbs. of sugar. A took 60 lbs. and B the rest. A needs 3 lbs. a week, B only 2. In how many weeks will both families have the same number of lbs. left?

9) To fill a bason, a laborer was employed with a bucket holding 3 gallons of water, which he empties 20 times in an hour; but the basin not filling fast enough, 8 hours later a second laborer is hired, with a bucket holding 4 gallons, which he empties as many times an hour as the first laborer empties his. In how many hours from the time the second laborer began to work, will both laborers have emptied the same quantity of water?

10) A gentleman who is now 48 years old, asked his son who is only 10 years old, How long must we live together, until I shall be only twice as old as you?

11) One of my acquaintances is now 40 years old, his son is 10 years. In how many years will this man, who is now just 4 times as old as his son, be only three times as old as he?

12) One of my acquaintances is now 30 years old; his

younger brother 20; that is, he is $\frac{3}{2}$ as old as his brother. In how many years will he only be $\frac{5}{4}$ of his brother's age ?

13) Two basins, of which one contains already 20 and the other 36 cubic feet of water, shall be further filled. Now it is known that each basin receives 2 cubic feet of water in one minute. In how many minutes will the first basin have $\frac{3}{4}$ as much water as the second ?

14) Two persons, A and B, set down to play. A had $100; B $40. Both of them won an equal sum of money; by which means A had just twice as much money as B. How much did each of them win ?

15) In a company of 80 persons, there are 3 times as many gentlemen as ladies; but the gentleman of the house assured them that he expected yet a number of other gentlemen with their ladies, after which there would be only twice as many gentlemen as ladies. How many gentlemen with their ladies did he expect ?

16) A had 3 times as much money as B, but when each had received an addition of $12, A had only twice as much as B. How much had each ?

17) A's fortune is $\frac{3}{4}$ as large as B's. Each of them gained $2,000; after which A's money is $\frac{4}{5}$ of B's. What was the fortune of each ?

18) I think of two numbers. One is $\frac{4}{5}$ of the other; but by subtracting 75 from each, the remainder of the larger is twice as great as the remainder of the smaller. What two numbers are those ?

c PROBLEMS IN WHICH TWO QUANTITIES ARE MADE EQUAL TO ONE ANOTHER, BY TAKING FROM THE ONE AND ADDING TO THE OTHER.

1) Two boys, A and B, played with nuts. A had 7 nuts, B had 3. How many nuts must A give to B, in order that both shall have the same number?

2) Two boys, A and B, played with apples. A had 100, B 50. How many times must A give B 5 apples to equal their number?

3) A and B play at billiards at $2 a game. A had $54, B $66. How many games must A win over B, in order to carry off as much money as B?

4) Two travellers, A and B, met on their journey. A had $300 with him, and B $500. A travels at the expense of the public, and receives daily $5, of which he spends only $2, adding the remaining $3 to the $300. B travels on his own account, and spends $2 a day from the $500 he has with him. When they had finished their journey, A had twice as much money as B. How many days were they journeying together?

5) At the battle of Leipzig 200,000 Germans, Russians and English opposed Napoleon's army, which was then composed only of 100,000 troops. During the engagement a number of Wirtembergians and Saxons, that were in Napoleon's army deserted his cause, and went over to the enemy; by which means he had exactly $2\frac{3}{4}$ times his own number to contend with. How many Wirtembergians and Saxons went over to the enemy?

6) A gentleman has in his draw 8 times as many crowns as sovereigns; but after taking away 400 crowns and adding 40 sovereigns, he found he had but 5 times as

many crowns as sovereigns. How many had he of each sort?

7) In a company there were, at first, 3 times as many gentlemen as ladies; but afterwards, when 8 gentlemen with their wives had gone away the proportion was still more unequal, for there remained 5 times as many gentlemen as ladies. How many gentlemen, and how many ladies were there in the company?

8) Saltpetre and sulphur are mixed together in a mass of 80 lbs., and in such a manner that for every 7 lbs. of saltpetre there are 3 lbs. of sulphur, (56 lbs. of saltpetre and 24 lbs. of sulphur;) how many lbs. of saltpetre must be *added*, and in its state be *taken away* from the sulphur, in order that the proportion of the mixture shall be as 11 to 4, or which means the same, that there may be $\frac{11}{4}$ as much saltpetre as sulphur?

9) A has 3 times as much money as B, but if A wins $15 from B, then A will have 4 times as much as B. How much money has each?

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d. PROBLEMS IN WHICH ONE QUANTITY IS MADE EQUAL TO ANOTHER BY ADDING OR SUBTRACTING.

1) The garrison of a fortress consists partly of infantry and partly of cavalry. Each horse soldier receives $8 a month, and each foot soldier $5. Now if the monthly pay of the whole garrison amounts to $1,800, how many horse, and how many foot soldiers are there in the fortress?

2) 20 persons, men and women, spent $21 at a tavern. The landlord told them that if each man was to pay him $1,25, and each woman, 75 cents, the whole would be paid. How many men, and how many women were there?

3) A mason, 12 journeyman, and 4 assistants, receive together $72 wages for a certain time. The mason receives $1 daily, each journeyman $\frac{1}{2}$ dollar, and each assistant 25 cents. How many days must they have worked for this money?

4) Twenty persons, masters and journeymen, spent $3,75 at an inn. The landlord told them that if each master would pay him $\frac{1}{4}$ dollar, (25 cents,) and each journeyman a New England shilling, ($\frac{1}{6}$ of a dollar) he should be paid. How many masters, and how many journeymen were there at the tavern?

5) A gentleman wishes to have 44 pieces, quarter dollars, and 4 pences for $5. How many pieces must he get of each kind of coin?

6) A father upon his return from a journey, distributes a number of nuts among his eight children, giving each child that was older than fifteen years 30 nuts, and to each that was younger than fifteen only 20. Upon counting they found that those that were older than fifteen had together received 90 nuts more than the others. How many children were there above, and how many under 15 years?

7) A capitalist has put out two different sums of money, amounting together to $1,000. One sum brings him 4 per cent, and the other 5 per cent interest; and yet he gets annually $4 more from the capital which is put out at 4 per cent than from the other which is put out at 5 per cent. What is the amount of each capital?

8) A master engaged a journeyman on the following conditions. For each day the journeyman is to work for him, he is to receive 75 cents; but for each day the journeyman should work for some other person, he is to pay him (the master) 25 cents for his board. At the end of

24 days they reckon with each other and find that they are just even. How many days did the journeyman work for his master, and how many days did he work for other persons?

9) 27 persons, men and women, received a certain sum of money; the men 70 cents apiece, and the women only 20 cents apiece; and yet the women received together as much as the men. How many men and women were there?

10) The sum of two numbers is 50; but when the greater is multiplied by 6, and the smaller by 5, the sum of the two products is 276. What are the two numbers?

11) The sum of two numbers is 36; but when the greater is multiplied by 4, and the smaller by 3, the difference between the two products is 32. What are the two numbers?

12) The sum of two numbers is 40; but when the greater is multiplied by 3, and the smaller by 5, the products are equal to one another. What two numbers are those?

D. Promiscuous Problems for the Exercise of the Learner.

[In the following problems no regularity has been preserved, in order to prevent the pupil from forming mechanical habits.]

1) On the first of January, 1796, a gentleman was asked his age. At the beginning of the 19th century, said he, I shall have lived a half a century. How old was he?

2) A thought his friend B was 50 years old. B told him that if he (B) had died 7 years ago, he would have died 50 years of age. How old was B ?

3) A gentleman dies, and according to his will leaves half of his property to his widow, $\frac{1}{3}$ to his son, and the remaining $1,000 to the poor. What was his property ?

4) I have two bags of money. In the one I have $\frac{1}{4}$, and in the other $\frac{3}{8}$ of my property. How much have I in each bag ?

5) A company consists half of lawyers, $\frac{1}{6}$ of physicians, and 15 ladies. How many lawyers and physicians were there in the company ?

6) A merchant sent his clerk travelling, and provided him with so much money that $\frac{1}{2}$ of it exceeded $\frac{1}{3}$ by $100. How much money did he give him for his journey ?

7) A gamester, after having lost $\frac{1}{6}$ and $\frac{1}{3}$ of his money, had yet $14 left. How much money had he at first ?

8) A shepherd drove himself $\frac{1}{2}$ of his flock to pasture, $\frac{1}{4}$ of them were driven by his son, $\frac{1}{8}$ by his daughter, and 40 remained in the stable. How many had he ?

9) The foot of a column is $5\frac{1}{2}$ feet high, the wooden shaft is $\frac{1}{2}$, the entablature $\frac{1}{5}$, and the gilded frame work $\frac{1}{7}$ of the whole height of the column. What is the height of the whole column ?

10) Two gentlemen, A and B, set out at the same time from two different places to meet each other. A makes 8 miles an hour, and B 5. The distance between the two places 260 miles. In how many hours will they meet ? And what part of the whole distance will each one have travelled ?

11) Two gentlemen, A and B, set out at the same time

to meet each other. A travels 8 miles an hour, and B 5. When they met, A had travelled 24 miles more than B. What was the original distance between A and B? And how far has each of them come?

12) A shopkeeper has a piece of cloth. After selling $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$, of it he has yet 9 yards left. How many yards were there in the whole piece?

13) A clerk was 6 years in the same house. In the first 3 years he spent only $300 a year, but in each following year $100 more than in the preceding year; and yet at the end of the 6th year he had saved $2,400. What was his salary?

14) A merchant paid a debt by selling to his creditor 2 cwt. of a certain commodity. As there was a difference in the quality, the creditor took 1 cwt. for $\frac{1}{2}$ the debt, and the other for $\frac{2}{3}$ of it. But the value of the commodity amounting now to more than the debt, the merchant received 8\frac{1}{3}$ back. What was the amount of the debt?

15) Two couriers that are 360 miles apart, are setting out in opposite directions. One travels at the rate of 5 miles and the other at the rate of 3 miles an hour. In how many hours will they meet?

16) A gentleman spent $\frac{1}{10}$ of his fortune in Holland, $\frac{1}{5}$ in France, $\frac{1}{8}$ in England, and $\frac{1}{4}$ in Italy. He has now but $975 left. How much money had he at first?

17) Four soldiers, an officer, a noncommissioned officer, a corporal, and a private took a chest of money from the enemy. The officer took $\frac{1}{4}$, the noncommissioned officer $\frac{1}{5}$, the corporal $\frac{1}{6}$, and the private $\frac{1}{8}$ of the contents, and the rest, consisting of $930, was delivered up to the regiment. What was the whole contents of the chest?

18) A company consisted $\frac{2}{5}$ of gentlemen, $\frac{1}{3}$ ladies, $\frac{1}{5}$ boys, and 12 girls. What was the number of the whole company? And how many gentlemen, ladies, boys, and girls were there?

19) Two persons, A and B, enter into partnership. A puts in \$10,000, B \$12,000. B's share of the gain is \$800 greater than A's. What is A's share? And what the whole gain?

20) A father has 6 sons, of whom each is 4 years older than his next younger brother, and the oldest is 3 times as old as the youngest. How old is the youngest son?

21) A boy was asked how many apples he had in his basket. He answered, if I had as many again, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ as many as I actually have, and yet 2 more, I should have 150. How many apples had he?

22) A and B played at billiards. A had \$42, B \$24. After a match, A was in possession of 5 times as much money as B. How much did A win?

23) A gentleman did not wish to tell his age; but his sister said that her brother and she were together 76 years old, and that she was 2 years older than her brother. What was the gentleman's age?

24) A person has two silver watches; one cost only $\frac{7}{8}$ as much as the other; but was on that account \$3 cheaper. What is the price of each?

25) The same gentleman (see problem 24) takes his two silver watches in order to exchange them for a gold one. The watchmaker told him that he would take one of the silver watches for $\frac{2}{5}$, and the other for $\frac{3}{8}$ of the price of the gold watch; and that therefore he must give him \$6$\frac{3}{4}$ to boot. What is the price of the gold watch?

26) How much money have you got? says A to B. If I had twice as much, answered B, and ½ as much lacking $6, I should have $50. How much money has B?

27) A cook receives from his master $11,25, for which he was to buy the same number of chickens, ducks, and geese. Each chicken costs 25 cents, (¼ dollar,) each duck 37½ cents, (⅜ dollar,) and each goose 50 cents, (or ½ dollar.) How many did he buy of each kind?

28) A gentleman said that ½ of his books were printed in Latin, ¼ were German, and the remaining 30 were French books. How many books had he?

29) A farmer asked another to give him half of his 100 geese. In order to have 100 geese, answered the other, I must have as many again, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{9}{10}$ as many as *you* in the bargain. How many had he?

30) A thief fled from a certain place, and walked 50 miles daily. 8 days later a constable was sent after him, who travelled 90 miles a day. In how many days will he have caught the thief?

31) Two Arabians were sitting under a tree to eat their dinner. One had 3 small loaves of bread and the other 5. A traveller passing by, asked permission to sit down and share with them; which being granted, they divided the 8 loaves equally among them. When the stranger rose he gave them 8 pieces of gold, of which one Arabian took 5 and the other 3. Did the two Arabians share the money in the right proportion? Why not? How many pieces, then, ought each to receive?

32) The 3d part of my income, says a person, I spend in board and lodging, the 8th part in clothes and washing, the 10th part in incidental expenses, and yet I save annually $318. What is the amount of his yearly income?

33) A gentleman bequeaths $\frac{4}{9}$ of his fortune to his friend, on condition that $\frac{3}{5}$ of this legacy should belong to his friend's children. The children received $60. What was the fortune of the gentleman ?

34) How old are you ? said A to B. B answered, If you add $\frac{1}{2}$ my age to my age, and subtract again $\frac{1}{3}$ of my age, there remain 70 years. What is B's age ?

35) Three sisters, A, B, and C, are together 72 years old. B is $2\frac{2}{3}$ years older than A, but $1\frac{2}{3}$ years younger than C. How old is each of them ?

36) A certain number of day laborers received 15 pence apiece; an equal number of others only 1 shilling apiece, by which means the former received together 2*s.* 3*d.* more than the latter. How many laborers were there of each kind ?

37) A has 6 times as much money as B, and yet A has only $45 more than B. How much money has each ?

38) Three thieves broke into a store to steal a chest of money, but the alarm being soon given, were obliged to flee, and left $516 on the counter. They were soon caught, when it was found that the first had $\frac{1}{9}$, the other $\frac{1}{7}$, and the third $\frac{1}{5}$ of the whole amount that had been in the chest. How much money was there in the chest ?

39) Two gentlemen, A and B, living 323 miles apart, set out, on the same day, to meet each other. A travelled 9 miles a day more than B, so that on the 19th they met on the road. How many miles did each travel a day ?

40) A, B, and C bought together 280 bushels of wheat; B bought $\frac{1}{2}$ as much as A, and C $\frac{1}{2}$ as much as B. How many bushels did each buy ?

41) A lady asked her mother whether she was not too

young to marry. O, no! answered the mother; for if I multiply the number of your years by $9\frac{3}{4}$ and subtract 12 from the product, there remain still 300 years. How old was the lady?

42) A schoolmaster asked his scholars to guess the number of dollars he had in his pocket. If I multiply this number by 3, said he, add 15 to the product, divide the answer by 6, and then add 6 to the quotient, I just obtain the number of dollars I have in my pocket. How many dollars had he?

43) I have a certain number in my thoughts, says A to B, try to guess it. I multiply it by 7, add 3 to the product, divide this by 2, subtract 4 from the quotient, and obtain 15. Now what number is it?

44) How much money have you about you? said a person to a friend. I have as many dollars about me, said the other, that if I multiply their number by 5, subtract 3 from the product, multiply this remainder again by 4, add 2 to the product, then take away the cipher to the right, (divide by 10,) I obtain $23. How many dollars had he?

45) One arithmetician desires another to find a number which he had in his mind. If, says he, you multiply it by 5, subtract 24 from the product, divide the remainder by 6, and add 13 to the quotient, I obtain this very same number. What number is it?

46) A father leaves 4 sons, A, B, C, D, and orders in his will, that A shall receive $100 less than $\frac{1}{2}$ his fortune, B $80 less than $\frac{1}{3}$, C $60 less than $\frac{1}{5}$, and D $\frac{1}{4}$ of his whole fortune. What was the whole amount of his fortune? And what was each son's share?

47) Two shepherds, A and B, counted the flock of their friend. I have only $\frac{1}{3}$ as many sheep, said A; I only

$\frac{1}{4}$ as many, answered B. Well, replied A, then I have just 17 more than you. What was the number of their friend's flock? And how many sheep had A and B?

48) A merchant striking the balance between his expenditures and his gain, finds that his expenses amount only to $\frac{2}{9}$, and his gain to $\frac{2}{7}$ of his fortune, and that consequently he had grown $400 richer. What is his fortune?

49) Alexander the Great said to his generals, I am 2 years older than Hephæstion. Clytus said, I am 4 years older than both. Calysthenes added, my father was 96 years old, and as old as all three. How old was Alexander? How old was Hephæstion? And how old was Clytus?

50) I am 30 years old, said a gentleman; my daughter is only $\frac{1}{2}$ year old. When will she be half as old as I?

51) I am now 50 years old; my son is 24. When will he be half as old as I?

52) Three persons, A, B, and C, are to share $1,200 in such a manner, that A shall receive $150 more than B, and B again $150 more than C. What is the share of each?

53) A young man owed a certain sum of money at different places. To the merchant he owed $\frac{1}{2}$, to the landlord $\frac{1}{5}$, to the tailor $\frac{1}{4}$, and to the shoemaker $\frac{1}{6}$ of the whole sum. His uncle paid the bills of the merchant, landlord, shoemaker, and tailor; but learned that the young man was still as much more in debt as he had paid for him. What was the amount of the whole debt?

54) A gentleman being asked the age of his son, answered, my wife is 23 years older than my son, I am as

old as my wife, and all three are 100 years old. What is the age of the son ?

55) Two vessels were sailing against each other ; one at the rate of 30 miles in 4 hours, the other at the rate of 11 miles in two hours. When they met one had sailed 36 miles further than the other. How many hours were they sailing to meet each other ? What was their original distance ? And how many miles did each of them sail ?

56) Three persons, A, B, C, loan together $400. B loaned $12 more than A, and C 16 more than B. What is the loan of each ?

57) A gentleman puts out two capitals at the same time. One of $500 at 4 per cent, and another of $600 at 5 per cent. In how many years will both together have gained $125 interest ?

58) $1170 are to be divided among three persons, A, B, and C, in proportion to their ages. B is $\frac{1}{3}$ older than A, but C is twice as old as A. What is the share of each ?

59) Four persons, A, B, C, D, are to share a sum of $400. D shall have $8 more than C, C $12 more than B, and B $16 more than A. What is the share of each ?

60) $48 are to be divided among 9 persons, and in such a manner, that each following person is to receive $\frac{1}{2}$ dollar less than the preceding. How much does each receive ?

61) A trader increased his fortune annually by $\frac{1}{3}$, but needs $1000 a year for the support of his family. At the end of 3 years he finds that he has just doubled his fortune. How much had he at first ?

62) Two gentlemen set out from two different cities at

the same time, to meet each other. A travels at the rate of 50 miles a day; B at the rate of 75 miles. In how many days will they meet?

63) 20 persons, men and women, spent 12*s.* at an inn; each man spent 8 pence, and each woman 7 pence. How many men, and how many women were there?

64) A grocer has a number of lbs. of a certain commodity. If he sells it at \$$4\frac{7}{8}$ per lb. he loses \$$2\frac{1}{2}$ on the whole; but if he sells it at \$5 per lb. then he gains \$10. How many pounds has he of the commodity?

65) A father dies and leaves a clear fortune of \$8,600, which his four sons, A, B, C and D, are to share in the following manner. A is to receive twice as much as B, less \$100; B is to receive 3 times as much as C, less \$200; but C 4 times as much as D, less \$300. How much does each receive?

66) A countryman carries eggs to market; but on the way 10 are stolen from him. Of the remainder he breaks exactly $\frac{4}{5}$ less 2 eggs. The few that he now has, he increases by buying 53 new ones; upon which he has but 11 eggs less than he had at first. How many had he at first?

67) A courier left 5 days ago, and travels at the rate of 6 miles an hour. Another is now sent after him, who travels at the rate of 8 miles. In how many days (or hours) will the second courier overtake the first?

68) A Greek woman went into the temple of Jupiter and prayed that he might double her money. The prayer being granted, she sacrificed to him 2 drachms. With the remainder of her money she now went to the temple of Apollo, and repeated the same prayer with the same

success; wherefore she sacrificed again 2 drachms. She now counted her money, and found that she had just twice as much as she had at first.

69) A regiment is marching from A to B at the rate of 3½ miles per hour. 8 hours later, another regiment sets out from B towards A, at the rate of 5⅙ miles. Now if the two places A and B are 80 miles apart, in how many hours, from the time the 2d regiment started, will these regiments meet?

70) A gentleman buys the same number of yards of two different kinds of stuff. Of the first he gets 8 yards for $3, and of the other 7 yards for $3. For the first he pays $8 less than for the second. How many yards did he buy of each stuff?

71) A capital is put out at 5 per cent interest, and at the end of 5 years $2,000 are received back for capital and interest. What was the amount of the capital?

72) A gentleman wants a capital at 4 per cent interest; which with the interest of 20 years shall amount to $900. What capital is it?

73) A number of soldiers are to share a reward. If each soldier receives $4, then one of them receives nothing at all; but if each receives $3, then there are $6 left. How many soldiers were there? And what was the reward to be divided among them?

74) A thief is running away from a certain place at the rate of 15 miles in 4 hours. 24 hours later, a constable is making after him at the rate of 20 miles in 3 hours. In how many hours will the thief be taken?

75) A tavern-keeper has 6 hogsheads of new, and 3 hhds. of old wine, which together cost $225. Of the old

wine, each hogshead costs $25 more than of the new. What is the price of each sort?

76) Two horsemen are 6,000 feet apart. One can make with his horse 800 feet in a minute; the other only 700. In how many minutes can these two horsemen meet?

77) If the distance between the two horsemen is still the same (6,000 feet); but the first horseman instead of riding against the other was to ride after him, in how many minutes would he overtake him?

78) The father of a family is now 61 years old, his youngest son is 13. In how many years will the father be only 3 times as old as his son?

79) A gentleman bequeathed in his will a certain sum to his domestics, which they are to share in the following manner. The valet de chambre is to receive $200, and $\frac{1}{2}$ of the remainder; of this remainder, the cook is to have $\frac{1}{5}$ and $400; and the $500 which still remain are to be given to the coachman. What was the amount of the whole legacy?

80) A person bought a number of yards of a certain stuff, at 15 pence a yard, and 8 yards more of another stuff at 10 pence a yard. He paid the same sum for both. How many yards did he buy of each kind?

81) A gambler increased his money in the first game by one dollar more than $\frac{1}{2}$; in the 2d game he increased the money he then had again by two dollars more than one half, and in the third game, the money he then had again by three dollars more than one half. He had now 4 times as much as at first. How much had he at first?

82) A woman wanted to buy a certain number of yards of riband. If she gave 1 shilling a yard she would lack 1*s.* 6*d.*; but if she gave 9*d.* a yard she would have 1*s.*

left. How many yards does she wish to buy? And how much money had she?

83) I once bought a considerable quantity of cloths, at the rate of 5 yards for $7, and sold it again at the rate of $11 for 7 yards, by which bargain I gained just $100. How many yards of cloth did I buy and sell?

84) A gentleman puts out a capital of $500 at 4 per cent interest. $3\frac{1}{2}$ years after, he puts another capital of $480 at 5 per cent interest. In how many years, after he put out the second capital, will he have drawn the same amount of interest from both capitals?

85) A merchant wishes to sell a piece of cloth containing 36 yards for $120; but the buyer cheapens down the price by as much as he afterwards pays for 4 yards. How much does he now pay for the whole piece?

[*Remark.* If the buyer pays as much less for the whole piece as he pays for 4 yards, he might have bought also, had he wished, $36+4=40$ yards for $120. Consequently 1 yard, &c.]

86) Three brothers were together 102 years old; each elder brother was 1 year older than twice the age of his next younger brother. What was the age of each?

87) 15 lbs. of coffee and 18 lbs. of sugar cost together 19*s.*, but 1 pound of coffee costs 2 pence more than 1 lb. of sugar. How many pounds were there of each commodity?

88) I am now 28 years older than my daughter, says a gentleman to his friend, and therefore just 5 times her age. How old was he? And how old was his daughter?

89) A trader has a certain capital which he doubles at the end of each year. But he spends at the end of the first year $800; at the end of the 2d year $1,600,

and at the end of the 3d $2,400, when he finds that he has spent his whole fortune. What was his capital at first?

90) A gentleman wanted to buy a farm for ready money. Having none on hand, he calls upon each of his debtors to pay him $200; but by this means he obtains $700 more than he needs. He is therefore asking them for only $150 apiece, upon which he finds that he still lacks $1,100 to the price of the farm. What was the price of the farm?

91) A wine merchant has two kinds of wine; the first kind is worth $2 a bottle, the second $1,50. He now wishes to make a mixture of 120 bottles for 10*s*. ($1\frac{2}{3}$ dollar) per bottle. How many bottles must he take of each kind for the mixture?

92) A courier who started 10 days ago from a certain place, is pursued by another from the same place, and by the same way. The first goes 40 miles every day, the other 90. In how many days will the 2d overtake the first?

93) But in what time will the 2d courier overtake the first, (problem 92) when it is supposed that the 2d starts 12 days later than the first, and his speed is to that of the first as 8 to 3?

94) A gentleman went into partnership with two different concerns, investing in the whole $10,000. In the one he gains 10 per cent, in the other 12 per cent. The whole annual gain from both amounts to $1,080. How much money has he in each concern?

95) A wine merchant has 3 casks. If the 2d is filled from the first, the first remains $\frac{2}{3}$ filled; if the third is filled from the first, there remain $\frac{5}{9}$; but when the first is to be filled from the 2d and 3d, then there are 30 gallons wanting. How many gallons does each cask contain?

96) Two farmers, A and B, possessed together 11 acres of ground; A 7 and B 4. One morning as they went out to plough them, they saw another husbandman with a plough just passing by them. Will you help us plough our fields, said the two farmers, A and B, and we will pay you 11 shillings for it? Willingly, said he, and they went and ploughed the 11 acres together. Supposing now, that the third husbandman has ploughed $\frac{1}{3}$ of the 11 acres; and that A and B have also each ploughed $\frac{1}{3}$ of them; how much is A and how much must B pay towards the 11*s*. wages?

97) A merchant in order to pay a debt is obliged to lower the price of a certain commodity to the prime cost, On account of bad book-keeping, he neither knows the weight nor the cost of it. He only remembers this much, that if he had sold it for $1,25 per lb. he would have gained $7,50, and that if he had sold it for 90 cents per lb. he would have lost $13,50 cents. What is the weight, and what the prime cost of the article?

98) Of two gamblers, A and B, A has 5 times as much money as B. A loses $16, B gains 24; upon which A has only twice as much left as B. How much money had each at first?

99) A gentleman has a capital of $24,000; part of it he puts out at 7 per cent interest, and the other part at 5. Of the first he draws $600 interest more than of the 2d. What are the two parts he put out at interest?

100) How happens it, asked one pedestrian of another, that you got 3,000 feet ahead of me; although I took twice as long steps as you? That I allow, answered the other, but I took 5 times as many steps as you. How many feet has each travelled?

101) Divide the number 46 into two such parts, that when the one divided by 7 and the other by 3, the quotients added together, shall make 10. Which are the two parts?

102) Find three numbers, that the 2d divided by the first gives 2 for a quotient and 1 for a remainder; the third divided by the second gives 3 for the quotient, with the remainder 3; and the sum of these three numbers shall be 70. What numbers are they?

103) 12 men and 8 women were hired to do a certain work for which they received £1. 7*s.* 4*d.* At another time 12 men and 11 women were hired in the same manner for £1 10*s.* 10*d.* What was the hire of a man? And what that of a woman?

104) I have two kinds of wine. The first costs $1,60 per gallon, the other only $1,00. I wish to make a mixture of 200 gallons at $1,20 per gallon. How many gallons must I take of each kind?

105) A father leaves to his children a fortune which they are to share *equally* among themselves. The first child receives $100 and $\frac{1}{10}$ of the remainder; the 2d $200 and $\frac{1}{10}$ of the remainder; the third $300 and again $\frac{1}{10}$ of the remainder, and so on. How large was the fortune, and how many children were there to share it?

Remark. Let $x =$ the whole fortune. Since the shares are all equal, $100 + \frac{1}{10}x = \frac{9}{10}x - 100 + 200$; whence $x =$ &c. The fortune divided by the share of a child gives the number of children.

106) From two different sized orifices of a reservoir, the water runs with unequal velocities. We know that the orifices are in size as 5 to 13; and the velocities of the

fluid are as 8 to 7. We know farther, that in a certain time there issued from the one 561 cubic feet more than there did from the other. How much water did each orifice discharge in that time?

Remark. If the orifices are in size as 5 to 13, then the smaller orifice is $\frac{5}{13}$ of the larger. If the velocities are as 8 to 7, then the velocity of the smaller orifice is $\frac{8}{7}$ of the larger; consequently the whole compound ratio of the size and velocity of the one orifice to that of the other, as $\frac{5}{13} \times \frac{8}{7}$ is to 1, or as $\frac{40}{91} : 1$, that is, to every cubic foot which issued from the larger orifice, $\frac{40}{91}$ cubic feet issue from the smaller; then if $x =$ number of cubic feet which issue from the larger orifice, $\frac{40}{91} x =$ number of cubic feet issuing from the smaller, and $x - \frac{40}{91} x = 561$, whence x $=$ &c.

(In the same manner may the two following problems be solved.)

107) A dog pursues a hare. Before the dog started, the hare had made 50 paces, and this is the distance between them at first. The hare takes 6 paces to the dog's 5; and 9 of the hare's paces are equal to 7 of the dog's. How many paces can the hare take, before the dog overtakes him?

108) Two bombardiers throw different shells from a battery. The first had already thrown 36 times before the 2d began to throw, and throws 8 times to the 2d's 7. On the other hand, the 2d uses as much powder for 3 of his throws, as the first does to 4. How many throws must the 2d make, in order to consume as much powder as the first?

109) Three gamblers, A, B, C, set down to play. A had £10 10*s*., B £6, and C had £5 10*s*. They agreed to stake their money together, and to share the gain, if any,

in proportion to their stakes. Meanwhile a fourth gambler came in and asked them to let him have one fourth share in the game; but as he had no money about him he would pay them if they should happen to lose, on the following day. They acceded to his proposition, and lost. On the following day the fourth gambler came and brought his $\frac{1}{4}$ of the stake. How is it to be divided among A, B, and C? And how much does each receive?

(Compare problem 31, page 120.)

110) The fortunes of four persons, A, B, C, and D, are determined by the following conditions: A and B have together $1,000; C and D $1,200; A's fortune is double of C's, and D's 3 times that of B. How much has each?

111) A gentleman has two purses. If he puts $8 in the first, it is only worth half as much as the second. But if he takes these same $8 and puts them in the 2d, then it is worth 3 times as much as the first. How much is in each purse?

112) Now, says a person, I spend $\frac{1}{7}$ of my income at the theatre, and the rest in my ordinary expenses; could I however receive an addition of $100 yearly, then I might spend $\frac{1}{5}$ of my income at the theatre, and still have $40 more than before for my ordinary expenses. What was his income?

Remark. Let $x =$ his income; then $\frac{6}{7}x =$ his ordinary expenses. If he received an addition of $100, his income would be $x + 100$. If he spent $\frac{1}{5}$ of it, he would have $\frac{4}{5}x + 80$ left; but as in this case he has $40 more left for his ordinary expenses, $\frac{4}{5}x + 80 = \frac{6}{7}x + 40$, whence $x =$ &c.

113) An admirer of the sciences, who until now has ex-

pended $\frac{1}{4}$ of his yearly income in the purchase of books, resolves, on account of an addition which he has recently received to it, to lay out $\frac{1}{3}$ of his income in this way; because he calculates that notwithstanding this increase, he will still have as much left for his ordinary expenses as before. What proportion did this addition bear to his former income?

Remark. In the first case he has $\frac{3}{4}$ of his income left for his ordinary expenses; in the 2d case only $\frac{2}{3}$. Hence if $\frac{1}{x}$ = the increase of his income, (his income being 1) $\frac{2}{3}$ of $\frac{1}{x}$ (the remainder of the increase) must make up (be equal to) the difference between $\frac{3}{4}$ and $\frac{2}{3}$ of his whole income; that is,

$$\tfrac{2}{3} \times \frac{1}{x} = \tfrac{3}{4} - \tfrac{2}{3}$$

$$\frac{2}{3x} = \tfrac{9}{12} - \tfrac{8}{12}$$

$$\frac{2}{3x} = \tfrac{1}{12} \text{ whence}$$

$$x = \&c.$$

(In the same manner may the following problem be solved.)

114) In a certain town each house-owner contributed formerly the 7th part of his house rent for taxes; afterwards the tax was raised, and each house-owner was obliged to pay the 6th part. How much must each house-owner raise the rent of his inmates in order that he may receive as much after paying the tax as he did before?

115) There were several children in a family. Upon the question how many there were, the eldest son answered, I have as many sisters as brothers; and I, replied the eldest daughter, have as many sisters as brothers. How many brothers, and how many sisters were there in the family?

116) A gentleman has two silver cups, and only one cover for them. One cup weighs 12 ounces *without* the cover, and *with* the cover twice as much as the other cup without the cover. The other cup weighs with the cover 3 times as much as the first. What is the weight of the cover? And what is the weight of the 2d cup?

117) A person possesses a wagon with a mechanical contrivance, by which the difference of the number of revolutions of the wheels on a journey may be determined. It is known that each of the fore-wheels is $5\frac{1}{4}$, and each of the hind wheels $7\frac{1}{8}$ feet in circumference. Now when in a journey, the fore-wheels have made 2,000 revolutions more than the hind wheels, how great was the distance travelled?

Remark. Let $x =$ number of revolutions of the hind wheels, then
$x + 2{,}000 =$ number of revolutions of the fore-wheels,
$7\frac{1}{8}\,x =$ number of feet made by the hind wheels,
$5\frac{1}{4}\,x + 10{,}500 =$ number of feet made by the fore-wheels,
and because the hind wheels must have come as far as the fore-wheels, $7\frac{1}{8}\,x = 5\frac{1}{4}\,x + 10{,}500$; whence $x =$ &c.

118) A and B possess together $570. If A's fortune were 3 times as large, and B's 5 times, they would together have $2,350. How much has each?

Let $x =$ A's sum,
then $570 - x =$ B's, &c.

119) A wine merchant has two kinds of wine. The one costs 9 shillings ($1\frac{1}{2}$ dollar) per gallon, the other 5. He wishes to mix both wines together, in such quantities, that he may have 50 gallons at 8 shillings per gallon. How much must he take of each sort to make up this mixture?

$x =$ the number of gallons of one,
$50 - x =$ the number of gallons of the other, &c.

120) Five gamesters have lost jointly \$$840\frac{5}{8}$; B's loss amounts to $\frac{1}{2}$ dollar more than triple A's; C's loss is \$2 less than twice B's; D lost $\frac{1}{4}$ dollar less than A and B together; and E twice as much as B, less $\frac{1}{8}$ dollar. How much did each of them lose?

121) A goldsmith has two kinds of silver, viz. one kind contains 14 parts of pure silver, (that is, each pound of it contains 14 oz. of pure silver, and 2 oz. of alloy) and the other 8. He cannot however use that which is as good as the first, nor that which is so bad as the 2d; because he wishes to make a dish, which shall be 20 lbs. in weight, each pound of it containing 12 oz. of pure silver. How many pounds must he take of each kind, so that by melting them together, he may obtain both, the desired weights and the desired alloy?

Remark. Let $x =$ number of lbs. of the first,
$20 - x =$ number of lbs. of the 2d,
then $14x =$ the amount of pure silver contained in the first,
$160 - 8x =$ the amount of pure silver contained in the second,
$20 \times 12 = 240 =$ the amount of pure silver to be contained in the mixture,
consequently, $14x + 160 - 8x = 240$,
hence $x =$ &c.

122) A wine merchant has 40 bottles of wine at 7*s.* a bottle; but deeming this price too high for his customers, he wishes, as he intends to deal fairly, to add as much water to it as will enable him to sell a bottle of the mixed wine for \$1. How many bottles of water must he add to it?

Let $x =$ number of bottles of water,
then $x + 40$, the number of bottles of wine at \$1,
$6x + 240$ shillings $=$ the price of the whole mixture,

which must be equal to the price of the 40 bottles at 7 shillings a bottle. The rest is sufficiently easy.

123) A person has 35 lbs. of silver, each of which contains 15 out of 16 parts pure, and he wishes to add so much copper to it that each lb. may contain only 12 parts pure silver to 4 parts alloy. How much copper must he add? (See problem 122.)

124) How much silver, 8 parts out of 16 pure, must be melted with $7\frac{1}{2}$ lbs. of 13 parts pure, that the contents may be brought to 9 parts pure?

125) A person wishes for 17 silver coins, consisting of four-pences and quarter-dollars, for $2,75. How many coins of each sort can he receive for that amount?

126) Saltpetre and sulphur are mixed together in a mass of 80 lbs. and in such a proportion that for every 7 parts of saltpetre there are 3 parts of sulphur. How much saltpetre must still be added to the mass so that the proportion of these substances may be as 11 to 4?

Remark. If, in 80 lbs. the proportion of saltpetre to the sulphur is as 7 to 3, then there must be 56 lbs. of saltpetre and 24 of sulphur. Now if $x =$ number of lbs. of saltpetre added, then $56 + x$ must be equal to $\frac{11}{4}$ of 24; whence the rest follows of course.

127) In a full wine cask there are three bung-holes; by the first the wine can be drawn off in 4, by the second in 6, and by the third in 8 hours. What time will it take, when all three bung-holes are spouting at once?

Remark. If the first bung hole draws off the whole wine in 4 hours, it will draw $\frac{1}{4}$ of it in one hour. For the same reason will the 2d bung-hole draw off $\frac{1}{6}$, and the

3d $\frac{1}{8}$ in one hour. All three bung-holes will therefore draw off $\frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{13}{24}$ of the whole cask. Now if $\frac{13}{24}$ are drawn off in one hour, how long before $\frac{24}{24}$ or the whole cask are drawn off?

$$\tfrac{13}{24} : \tfrac{24}{24}\ 1 : x,$$

$$\text{or } 13 : 24 = 1 : x, \text{ whence } x = \&c.$$

(In a similar manner may the two following problems be solved.)

128) A cistern can be filled by three pipes; by the first in $1\frac{1}{3}$ hours; by the 2d in $3\frac{1}{3}$ hours; and by the 3d in 5 hours. In what time (how many minutes) will the cistern be filled when all these pipes are opened at once?

Remark. The first fills the cistern in $1\frac{1}{3}$ hours, or 80 minutes; it will therefore fill $\frac{1}{80}$ of the cistern in 1 minute; the 2d, &c.

129) Three masons are employed in building a wall. The first does 8 cubic feet in 5 days, the 2d 9 cubic feet in 4 days, and the third 10 cubic feet in 6 days. How much time (days) will these masons need, when they work together, to build 756 feet of the wall?

Remark. If the first does 8 cubic feet in 5 days, he does $\frac{8}{5}$ in one day. For the same reason does the 2d build $\frac{9}{4}$, and the 3d $\frac{10}{6} = \frac{5}{3}$ in one day. The rest of the solution is as in the preceding problems.

130) A person has 3 pieces of metal of equal size. Of the first 5 cubic inches weigh $69\frac{3}{4}$ ounces; of the 2d, $3\frac{1}{3}$ cubic inches weigh 41 ounces, and $4\frac{2}{7}$ cubic inches of the third weigh 91 ounces. The three pieces together weigh $949\frac{2}{3}$ ounces. What is the size (how many cubic inches) of each?

Remark. Since 5 cubic inches of the first weigh $69\frac{3}{4}$ ounces, 1 cubic inch weighs $69\frac{3}{4} : 5 = 13\frac{19}{20}$. For the same reason does 1 cubic inch of the second weigh $41 : 3\frac{1}{3} = 41 : \frac{10}{3} = \frac{123}{10} = 12\frac{3}{10}$ ounces, and 1 cubic inch of the 3d, $91 : 4\frac{2}{7} = 91 : \frac{30}{7} = \frac{637}{30} = 21\frac{7}{30}$ ounces. Now if $x =$ number of cubic inches of each piece, $12\frac{3}{10}x + 12\frac{3}{10}x + 21\frac{7}{30}x = 949\frac{2}{3}$; whence $x =$ &c.

131) A charitable man wished to collect some money in a numerous company for a poor person. Each of the company offering to contribute 75 cents, the collector considered it too much, because he would then collect $25 more than was required for the man. He therefore proposed that each of them should only give 30 cents; but on calculation, found that it would be too little; because he would then fall short of the required sum by $20. How many persons did the company consist of? How much does the man need? and how much must each contribute to collect the sum required?

132) A person wishes to raffle away a gold watch, and for that purpose makes a number of tickets. If he sells the tickets for $1,25 apiece, he would lose $20, because the watch costs more than he would in this case get for all the tickets; but if he sells a ticket for $1,62½, then he gains $10. How much therefore does the watch cost him? And how many tickets must he sell?

133) How old am I? asked a son of his father. The father answered, your age is now just ⅓ of mine; but 6 years ago it was only ¼ of my age. How old is the father now? And what is the age of his son?

134) In order to meet all my expenses, said a person, I ought to have a yearly income of $540; but it wants a considerable sum of this. Were my income 3½ times as large as it really is, then I should not only be able to meet my expenses, but even have as much above my expenses as I am now lacking towards them. What was this person's yearly income?

Remark. Let $x =$ his income ; then
$3\frac{1}{2}x - 540 =$ what his income would exceed his expenses, if it were $3\frac{1}{2}$ times greater ;
$540 - x =$ what his income is now lacking toward his expenses.

The rest of the solution follows of course.

135) A and B have together a fortune of $9,800. A puts out $\frac{1}{6}$ of his money, B only $\frac{1}{5}$ of his ; and yet they have the same sum of money remaining on hand ? What is each man's fortune ?

136) A copyist was asked how many sheets he wrote a week. Not yet 70, was the answer ; because I work but 4 hours a day. But were I to work 10 hours a day, then I should write weekly as much above 70 sheets, as I am now deficient. How many sheets did he write weekly ?

Remark. Let $x =$ number of sheets written in 1 hour, then $70 - 28x =$ number of sheets he is deficient in a week, when working but 4 hours a day, and $70x - 70 =$ number of sheets above 70 when working 10 hours a day ; from which the rest follows of course.

137) A and B owe together $29. Neither of them can pay the debt alone. For this reason, says A to B, give me $\frac{2}{3}$ of your money and I shall pay the debt. No, replied B, give me rather $\frac{3}{4}$ of yours and I will pay the debt. How much money has each ?

Remark. Let $x =$ B's money ; then because $\frac{2}{3}$ of B's money must be *added* to A's money to make 29 dollars, A's money must be the *difference* between 29 and $\frac{2}{3}$ of B's, that is, $29 - \frac{2}{3}x =$ A's money ; whence $\frac{3}{4}$ of A's money $= \frac{87}{4} - \frac{2}{3} \times \frac{3}{4}x = \frac{87}{4} - \frac{1}{2}x$, which added to B's money (x) must be equal to the whole debt of $29. Thus we have the Equation, &c.

138) A gentleman buys three different kinds of cloth. The first is \$2½ a yard, the 2d \$2, and the third \$1 a yard. Of the best cloth he takes only half as many yards as of the second quality, and of that only half as many as of the third. He pays \$52½ for the whole. How many yards did he buy of each ?

139) Four persons, A, B, C, and D, have won together \$1,170. If A multiplies his money by 3, B by 4, C by 5, D by 6, the products are all equal to one another. How much did each win ?

Remark. Let $x =$ D's gain, then $6\,x = 5\,C$
or $\frac{6}{5}\,x =$ C's,
$\frac{5}{4} \times \frac{6}{5}\,x = \frac{3}{2}\,x =$ B's,
and $\frac{4}{3} \times \frac{3}{2}\,x = 2\,x =$ A's gain, whence the rest follows of course.

140) A person asked a surveyor how far the two signals, whose distance he had just measured, were from one another. The surveyor answered, their distance is far short of 1,000 feet; for if I add to the distance its third part, increase this sum by 176, and multiply this sum again by 2½, then I shall as much exceed 1,000, as the distance itself falls now short of it. How many feet were the two signals from one another?

(Compare problems 134 and 135.)

141) Find a number which has this property, that if it be multiplied by 5, it gives a product which is as much above 20, as the number itself is under 20. What number is it ?

142) A farmer employed two journeymen, who work for him at the same rate. To one he gave 16 bushels of rye and \$14, for 56 days' work; and to the other 30 bushels

of rye and \$17¼ for 84 days' work. What did he calculate a bushel of rye ?

Let $x =$ number of cents a bushel of rye costs, then
$16\,x =$ number of cents 16 bushels cost, and
$1{,}400 + 16\,x =$ the wages for 56 days ;
consequently $\frac{1{,}400 + 16\,x}{56} = 1$ day's wages.

In the same manner calculate 1 day's wages of the 2d laborer; and as they have received *equal* wages, it will not be difficult to state the equation.

143) I once had an untold sum of money lying before me. From this I first took away the third part and put in its stead \$50. A short time after I took from the sum thus augmented the 4th part, and put again in its stead \$70. I then counted my money, and found \$120. What was the original sum ?

144) From a sum of money, \$50 more than one half is first taken away ; from the remainder \$30 more than its fifth part ; and again from the 2d remainder \$20 more than its 4th part. At last there remained only \$10. What was the original sum ?

Remark. Observe that $\frac{1}{2}\,x - 50$ less $\frac{1}{10}\,x - 10 + 30$ are equal to $\frac{4}{10}\,x - 40 - 30 = \frac{2}{5}\,x - 70$.

From $\frac{2}{5}\,x - 75$, you must again take one fourth and 20; which will then be equal to 10.

145) A countryman brings eggs to market, and first sells 4 more than one half of them ; then he goes further and sells half of the remainder and 2 over ; 6 eggs more than half of the remainder are now stolen from him, and dissatisfied about this loss he returns to his village with the 2 eggs which remained in his basket. How many eggs did he take to town ?

146) A capitalist engages to lend a merchant $16,000 for 15 months; but as he is not able to furnish this sum at once they agree to this, that the capitalist shall first furnish $5,000; after the expiration of 6 months $3,000 more; and then in eight months the last $8,000. How long after receiving the last $8,000 can the merchant retain the whole capital of $16,000, so that neither party shall suffer?

Remark. This problem is similar to what merchants call an equation of payment.

Let $x =$ number of months he is to retain the last $8,000, then $x + 8$ is the number of months he retains the $3,000, and $x + 14$ is the number of months he retains the $5,000, (because he has received $3,000 8 months, and $5,000 14 months sooner than the $8,000.) Hence the whole capital of $16,000, multiplied by the time of 15 months, must be equal to 8,000 multiplied by x months, + $3,000 multiplied by $x + 8$ months, and $5,000 multiplied by $x + 14$. The rest of the solution needs no further explanation.

147) A land proprietor entered into a contract with his neighbor, in which he bound himself to take 500 of his neighbor's oxen into pasture for 1 year. The neighbor however sent, with the land owner's consent, at first only 200 head; in 7 months 250 more, and in 8 months after that again 50 more. How much longer must the land owner feed all the 500 oxen to fulfil as much as his original engagement?

(This problem is solved like the preceding one.)

148) A person purchases a certain commodity for $4,500, which he is to pay for at the expiration of a year. He agrees subsequently with the seller to give him $1,500 cash, and the remaining 3,000 at four equal periods, $750

each time. What periods must be fixed upon so that neither party shall be a loser ?

Remark. Let $x =$ the number of months these periods are apart,

then the first $750 are due in x months,
the 2d $750 are due in $2x$ months,
the 3d $750 are due in $3x$ months,
the 4th $750 are due in $4x$ months,

and these four capitals multiplied by their respective times, must equal the $4,500 multiplied by 12 months.

149) A certain sum is to be paid as follows: $1,376 in 5 months ; 3 months later $2,560, and the remainder 5 months after this If the whole sum be paid at one single instalment, it must be paid in 10 months. What is the sum to be paid ?

Remark. $x =$ the capital,
than $1,376 multiplied by 5,
$+$ $2,560 multiplied by 8,
and $3,936 multiplied by 13,
must equal the whole capital by its time.

150) A person is to pay $7,000, viz. $2,000 in $3\frac{1}{2}$ months, $3,500 in 4 months, and $1,500 in 14 months. His creditors propose to him to pay this sum in two different instalments, the half each time, so that the 2d payment may be 1 month later than the first. If the debtor agrees to this, in what time must the first instalment be paid ?

Remark. The sum of the three first instalments, each multiplied by its respective time, must be equal to the sum of the two instalments of $3,500 each, multiplied by its respective time x, and $x + 1$.

151) Two persons, A and B, bid for a farm which is to be sold. The first offers \$7,705; viz. \$3,365 cash, and \$4,340 in 8 years; or if the seller wishes it, he will give the \$4,340 also in cash, deducting however \$5 per cent discount. The 2d bids another sum, which, calculated from this time, is to be paid in three equal instalments; each at an interval of 2 years; viz. the first in 2, the 2d in 4, and the 3d in 6 years; or if the seller wish it, cash immediately, deducting 5 per cent discount on each instalment. The seller considers both offers the same. How much did the second bid?

Solution. If the seller accepted the first proposition, he would receive \$3,365 + \$3,100 = \$6,465; (because \$4,340 payable in 8 years, are equal to \$3,100 cash, if discount be calculated at the rate of 5 per cent; which may be found from the proportion 140 : 100 : : 4,340; Answer, = 3,100.) *

Now if $x =$ the offer of the 2d, then the cash of the first, $\frac{1}{3}x$ is found by the proportion 110 † : 100 : : $\frac{x}{3}$;

$$\text{Answer} = \frac{100x}{330};$$

the cash of the 2d $\frac{1}{3}x$ is found by 120 : 100 : : $\frac{x}{3}$;

$$\text{Answer} = \frac{100x}{360};$$

and the cash of the 3d $\frac{1}{3}x$ is found by 130 : 100 : : $\frac{x}{3}$:

$$\text{Answer} = \frac{100x}{390};$$

and as the whole cash of the 2d offer must be equal that of the first $\frac{100}{330}x + \frac{100}{360}x + \frac{100}{390}x = 6{,}465$; whence $x =$ &c.

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* We say in this proportion 140 is to 100, because \$100 at 5 per cent interest are in 8 years equal to \$140.

† Because \$100 gives in 2 years \$10 at the rate of 5 per cent.

152) Three merchants, A, B, and C, enter into trade. A contributes $1,200, B $500, and C $600. A leaves his money 8 months, B 10 months, and C 14 months in business. They gain $500. What is the share of each?

Remark. This is nothing else than what in Arithmetic is called *Compound* Fellowship. The shares must be proportioned to the money each put in trade, multiplied by the time he suffered it to remain there.

153) Three persons, A, B, and C, contribute a certain sum for business. B contributes $\frac{1}{2}$ more than A, and C $300 more than A and B together. After some time the profit, amounting to $5,020, is divided, and C receives $2,570 for his share. How much did each contribute?

Remark. Let $x =$ what A contributes,
then $\frac{3}{2}x =$ what B contributes,
then $\frac{5}{2}x + 300 =$ what C contributes,
the whole stock is $\overline{5x + 300}$,
and as the whole stock is to A's, so is the whole gain to A's; from which proportion find the x (making the product of the means equal to that of the extreme). Having found A's share, the whole is reduced to a problem in simple fellowship.

154) Three merchants, A, B, and C, form themselves into a trading company; C contributed $5,600; but A $320 less than B. A leaves his money 7 months, B 14 months, and C 12 months in trade. The profits amounting to $2,402$\frac{1}{6}$ are divided among the partners, in proportion to the sums advanced and the time they remained in trade. Now B receives 879\frac{2}{3}$ for his share. What did A and B contribute?

Remark. This problem is similar to the preceding one only that *the contribution of each partner must yet be multiplied by his time.*

$$\left\{\begin{array}{l} x = \text{what A contributes,} \\ x + 320 = \text{what B contributes,} \\ 5{,}600 = \text{what C contributes.} \end{array}\right\}$$

155) A person dies and leaves 4 sons and a fortune of $1,100. Ten months after his death the will was first opened, and in this time the children had expended all the property they had inherited, together with the interest of it. At the same rate of expenditure, and the same rate of interest did 3 other children spend a capital of $1,200 in 15 months. How high were the interests of these sums calculated? And how long would it take 6 children under the same circumstances, to expend $1,650?

Solution. Let x = the rate of interest, (number of per cent,)

then $\frac{10}{12}$ of $\frac{x}{100}$ of $1{,}100 =$ interest of $1,100 for 10 months,

and $\frac{15}{12}$ of $\frac{x}{100}$ of $1{,}200 =$ interest of $1,120 for 15 months,

but $\frac{10}{12} \times \frac{x}{100} \times 1{,}100 = \frac{110}{12}\, x$,

and $\frac{15}{12} \times \frac{x}{100} \times 1{,}200 = 15\, x$;

therefore $1{,}100 + \frac{110}{12}\, x =$ the whole amount spent by the 4 sons;

and $275 + \frac{110}{48}\, x =$ what 1 son spent in 10 months;

whence $27\frac{1}{3} + \frac{11}{48}\, x =$ what 1 son spent in 1 month.

In a similar manner we find that

$400 + 5\, x =$ what a child spent in 15 months,

or that $26\frac{2}{3} + \frac{1}{3}\, x =$ what a child spent in 1 month;

and as the rates of expenditure are the same,

$$27\tfrac{1}{2} + \tfrac{11}{48}\,x = 26\tfrac{2}{3} + \tfrac{1}{3}\,x\,; \text{ whence } x = \text{\&c.}$$

The rest is sufficiently easy.

156) Five brothers, in the space of 9 months, squandered a capital of $4,800, together with the interest of it for this whole time. At the same rate of expenditure, 2 other persons squandered $3,320 with interest, in 16 months. The rate of interest was in both cases the same. How much did each spend monthly ?

Remark. Find first the rate of interest as in the preceding problem ; from which the rest is easily determined.

157) A farmer brought eggs to market and offered them for sale at 7 cents a dozen. A person who was passing by stumbled against the basket and broke 5 dozen of his eggs. Being paid for them, he resolved to sell the remainder at 8 cents a dozen ; because he would then get as much by them as he would have received before for his full number ; besides the money for the eggs that were broken, which would then be clear profit ?

Remark. Supposing $x =$ number of dozen he had at first,

then $x - 5 =$ what remained,

and x dozen at 7 cents equal $x - 5$ at 8 cents a dozen, because the money he receives for the broken ones is clear profit.

158) A cook carrying lemons, was asked how many he had. As he was a good arithmetician he answered in the following manner : these lemons cost me 75 cents a dozen, but had I got the 5 lemons into the bargain, which I asked for, they would have cost me $10\frac{5}{12}$ cents less per dozen. How many lemons did he carry ?

Let $x =$ number of lemons ;

then $\frac{x}{12}$ number of dozen ;

and $\frac{75\,x}{12}$ = price of them.

In the 2d case, where he obtains 5 more,

$\frac{x+5}{12}$ = number of dozens; and as in this case they are $10\frac{5}{12}$ cheaper than 75 cents a dozen,

$$\frac{x+5}{12} \times 74\tfrac{7}{12} = \text{the price of them,}$$

which must be equal to the first price, $\frac{75}{12}\,x$; the rest is sufficiently easy of itself.

159) A merchant adds yearly to his capital one third, but takes from it, at the end of each year, $1,000 for his expenses. At the end of the 3d year, after deducting the last $1,000, he finds himself in possession of just twice the sum he had at first. How much did he possess at first?

160) A reservoir can be filled by three pipes. The first and 2d fill it in 2 hours, the first and third in 3, the 2d and 3d in 4 hours. In how many hours will all three fill it?

Let the pupil observe that the 1st and 2d together fill in 1 hour $\frac{1}{2}$, the 1st and 3d together $\frac{1}{3}$, and the 2d and 3d together $\frac{1}{4}$, of the whole reservoir, and that 1st and 2d, 1st and 3d, 2d and 3d, make together just twice the amount of the three pipes, (the 1st, 2d, and 3d being counted twice). The three pipes together will therefore fill $\frac{1}{2}$ of $(\frac{1}{2}+\frac{1}{3}+\frac{1}{4})$ in 1 hour; whence the rest follows of course.

161) A merchant increases his capital yearly by 20 per cent; but takes from it every year $1,000 for the support of himself and family. After he had carried on his business in this manner for 3 years, he finds, after deducting the usual sum of $1,000, that his capital has increased

$200 more than $\frac{3}{5}$ of the original sum. What was his original sum ?

162) I take a certain number, multiply it by $3\frac{3}{7}$, take 60 from the product, multiply the remainder again by $2\frac{1}{2}$, and subtract again 30, when nothing remains. What is the number ?

163) A general wishing to draw up his regiment into a square, tried it in two ways. The first time he had 39 men over ; the 2d, having extended the side of the square by 1 man, he wanted 50 men to complete the square. What was the number of soldiers in the regiment ?

Remark. Let $x =$ number of soldiers he first put in the side of the square;
then $x^2 + 39 =$ the number of his regiment.
If he increases the side by 1 man, then
the side is $x + 1$
which multiplied by $x + 1$
gives $x^2 + 2x + 1$;
but as in this case he wants 50 men,
$x^2 + 2x + 1 - 50 = x^2 + 2x - 49$
is also the number of his regiment ;
hence $x^2 + 39 = x^2 + 2x - 49$; or $39 = 2x - 49$,
whence $x =$ &c.

Having found x, raise it to the square, and add 39, and you have the number of the regiment.

164) A person has a number of dollars, which he wishes to arrange in the form of a square. At the first trial he had 130 dollars over, but when he enlarged the side of the square by 3 dollars, there only remained 31. How many dollars had he ?

(This problem is similar to the preceding one)

165) A butcher bought a number of oxen. Half of

them he puts out to pasture, $\frac{1}{3}$ of them he keeps in the stable, $\frac{1}{12}$ of them he kills, and the rest he sells, each ox for as many dollars as he sells oxen; wherewith he gains as many dollars as he bought oxen at first. How many oxen did he buy?

Remark. When all the terms of an equation contain the unknown quantity, then the whole equation may be divided by x. This is the case with the equation obtained from this problem.

166) Determine the size of three wine casks from the following conditions: when the first empty cask is filled from the second full one, there only remains in the second $\frac{2}{9}$ of the wine that was in it; if the second cask is filled from the third full one, then there remains in the third only $\frac{1}{4}$ of the wine that was in it; but if we wished to fill the 3d empty cask from the first full one, there would not be enough wine by 50 gallons. How many gallons does each cask contain?

Remark. Let $x =$ the number of gallons of the 3d cask,
then $\frac{3}{4}x =$ number of gallons of the 2d,
and $\frac{7}{9}$ of $\frac{3}{4}x =$ number of gallons of the first.

167) A person has four wine casks of different sizes. When he fills the second empty cask from the first full one, there remains in the first only $\frac{4}{7}$ of the wine; when he fills the 3d empty cask from the 2d full one, then there is left in the 2d only $\frac{1}{4}$ of the wine; but when he attempts to fill the 4th empty cask from the third full one, then only $\frac{9}{16}$ of the 4th is filled, and if he wishes to fill the 3d *and* 4th empty casks from the first full one, then they would not only be filled, but he would have 15 gallons over. How many gallons then does each of these four casks contain?

Remark. Let x = the number of gallons of the 4th cask

then $\frac{9}{16}x$ = the number of gallons of the 3d,

$\frac{4}{3}$ of $\frac{9}{16}x = \frac{3x}{4}$ = the number of gallons of the 2d

because if the 3d cask is $\frac{3}{4}$ of the 2d, the 2d must be $\frac{4}{3}$ of the 3d. For the same reason is the first cask $\frac{7}{3}$ of the 2d, because the 2d is filled with $\frac{3}{7}$ of the first. The rest of the solution is easily found.

SECTION X.

Problems for Simple Equations with two or more unknown Quantities.

1) Required two numbers whose sum is 70, and whose difference is 10?

Remark. Let the two numbers be represented by x and y.

2) Two purses together contain $300. If you take $30 out of the first and put them into the second, then there is the same in each. How many dollars does each contain?

Remark. This and similar problems may also be solved with *one* unknown quantity; but the beginner will find it more easy to use two.

3) A gentleman has two purses. If he puts $10 out of the first, then there is twice as much money in it as in the 2d. But if he puts $10 into the 2d, then the same sum of money is contained in both. How much money is in each purse?

4) A says to B, give me $100 and I shall have as much as you. No, says B, give *me* rather $100, and then I shall have twice as much as you. How many dollars had each ?

5) A person has two snuff-boxes. If he puts $8 into the first, then it is half as valuable as the other ; but if he takes the $8 out of the first and puts them into the 2d, then this is worth 3 times as much as the first. What is the value of each box ?

6) A and B possess together a fortune of $570. If A's fortune were 3 times and B's 5 times as large as each really is, then they would have together $2,350. What is the fortune of each ?

7) Two persons, A and B, were eating apples. Says A to B, if you give me 2 of yours, I shall have as many as you ; if you give me 2 of yours, answered B, I shall have twice as many as you. How many had each ?

8) Find two numbers of the following properties : when the one is multiplied by 2, and the other by 5, and both products added together, the sum is 31 ; on the other hand, if the first be multiplied by 7, and the 2d by 4, and both products added together, the sum $=68$.

9) Two numbers are given by the following properties. If the first be increased by 4, it is $3\frac{1}{4}$ times as great as the second ; but if the 2d be increased by 8, then it is half as great as the first. What are the two numbers ?

10) Said a man to his father, how old are we ? Six years ago, answered the latter, I was one third more than 3 times as old as you ; but three years hence I shall be obliged to multiply your age by $2\frac{1}{6}$ in order to obtain my own. What is each one's age ?

11) A and B jointly have a fortune of $9,800. A invests the sixth part of his property in business, and B the 5th part, and still each has the same sum remaining. How much has each?

12) A and B owe together $600; but neither has money enough to pay this debt alone. For this reason, says A to B, give me $\frac{3}{4}$ of your money, and I can pay the debt; B says to A, give me only half of your money and I can pay it. How much money has each?

13) A owes $1,200, B $2,500; but neither has money enough to pay his debt. Lend me, says A to B, the 8th part of your money, and I shall be enabled to pay my debt. B answered, I can discharge my debt if you lend me the 6th part of yours. What was the fortune of each?

14) Two gamblers, A and B, lost an equal sum of money. A lost $\frac{1}{4}$, B $\frac{1}{5}$ of the money he had about him. Both together lost $8. How much money had each about him.

15) Two persons, A and B, played for money. A lost $\frac{1}{3}$, and B $\frac{1}{4}$, of the money he had about him, and yet each had the same sum left. Both together lost $10. How much money had each at first?

16) A capitalist borrows $8,000 on favorable conditions, because he has an opportunity of putting out $23,000 at higher interest; and he has an overplus of $905 of interest yearly. On the same conditions he borrows, on the one hand, $9,400, and puts out on the other, $17,500; this brings him an overplus of $539½ interest yearly. At what rate of interest did he borrow and lend money?

Remark. Let $x =$ the number of per cent he pays for the money he borrows,

$y =$ the number of per cent he received for the money he lends,

$$\left.\begin{array}{l}\text{then } \frac{x}{100}\text{ of } 8{,}000 = 80\,x = \text{the interest he pays,}\\ \text{and } \frac{y}{100}\text{ of } 23{,}000 = 230\,y = \text{the interest he receives,}\end{array}\right\}\text{on the first capital,}$$

consequently, $230\,y - 80\,x = 905$.

In a similar manner may the 2d equation be obtained from the 2d statement of the problem. The rest is sufficiently easy of itself.

17) A person has two large pieces of iron. It is known that $\frac{2}{5}$ of the first piece weigh 96 lbs. less than $\frac{3}{4}$ of the second; and that $\frac{5}{8}$ of the second weigh exactly as much as $\frac{4}{9}$ of the 1st. What is the weight of each piece?

18) A cistern containing 210 buckets may be filled by two pipes. By an experiment in which the first was opened 4, and the 2d 5 hours, 90 buckets of water were obtained. By another experiment in which the first was open 7, and the other $3\frac{1}{2}$ hours, 126 buckets were obtained. How many buckets does each pipe discharge in 1 hour? And in what time will the cistern be filled when the water flows from both pipes at once?

Remark. Find first how many buckets each pipe discharges in 1 hour. The 2d question is solved by a simple proportion, similar to that given in the remark to problem 127 page 137.)

19) A person has 50 coins, consisting of sovereigns and shillings; their value amounts to £334 14*s*. How many has he of each kind?

20) A cook bought once 4 oranges and 4 lemons for 28 pence. In the same manner he bought at another time 4 oranges and 5 lemons for 33 pence. What was the price of an orange? And what that of a lemon?

21) A trader wanted to pay a debt of \$30 with 60 lbs. of coffee, and 40 lbs. of sugar. He calculated 4 lbs. of sugar as high as 3 lbs. of coffee. What is the price of 1 lb. of each commodity?

22) A person has two kinds of goods. 8 lbs. of the first and 9 lbs. of the 2d cost together \$18,46. Farther, 20 lbs. of the first, and 16 of the 2d, cost together \$36,40. What is the price per lb. of each article?

23) 15 Silician and 33 Leipzig ells together are equal to $39\frac{1}{2}$ Brabant ells; further, 24 Silician ells and 55 Leipzig ells, are together equal to 65 Brabant ells. What proportion, according to these conditions, do the Silician and Leipzig ells bear to the Brabant? And what proportion does the Silician ell bear to 1 Leipzig ell?

Remark. Let $x =$ the length of 1 Silician ell expressed in Brabant measure,
$y =$ the length of the Leipzig ell expressed in Brabant measure.

24) 40 French miles, when reduced to German miles, amount to $12\frac{1}{2}$ such miles more than 53 English; 10 French and $26\frac{1}{2}$ English miles are together equivalent to $11\frac{3}{4}$ German miles. At this rate, what proportion do the French and English bear to the German miles? And what relation have the French to the English miles?

Remark. Let $x =$ the length of a French mile expressed in German measure,
$y =$ the length of an English mile expressed in German measure.

Then
$$\left.\begin{aligned} 40\,x &= 53\;y + 12\tfrac{1}{2} \\ 10\,x + 26\tfrac{1}{2}\,y &= 11\tfrac{3}{4} \end{aligned}\right\}\text{or}\ \begin{aligned} 40\,x - 53\;y &= 12\tfrac{1}{2}, \\ 10\,x + 26\tfrac{1}{2}\,y &= 11\tfrac{3}{4}, \end{aligned}$$
whence, &c.

25) A person has two horses and two saddles, one of which cost \$50, and the other \$2. If he places the dear saddle on the first horse, and the cheap one on the 2d, then the 2d horse and saddle are together worth \$8 less than the first horse and saddle; but if he puts the cheap saddle on the first horse and the dear one on the 2d, then the 2d horse and saddle are worth $3\frac{3}{4}$ times as much as the other horse and saddle. What is the value of each horse?

26) There is a fraction such, that if 1 be added to its numerator, its value is $= \frac{1}{3}$; but if 1 be added to the denominator, the value of the fraction $= \frac{1}{4}$. What fraction is it?

Remark. Let $x =$ the numerator,

$y =$ the denominator,

$\frac{x}{y} =$ the fraction.

Then $\frac{x+1}{y} = \frac{1}{3}$, &c.

27) It is required to find a fraction such, that if 3 be subtracted from the numerator and denominator, it is changed into $\frac{1}{4}$; but if 5 be added to the numerator and denominator, its value is changed into $\frac{1}{2}$. What fraction is it?

28) B has lent out at interest \$12,600 more than A, and obtains 1 per cent more for his money; on account of which his interest amounts yearly to \$730 more than A's. C has lent \$3,000 more than A, and at 2 per cent higher interest, wherefore he obtains for his sum \$380 yearly interest more than A. How much has each lent? And at what interest?

Remark. Let $x =$ A's capital;

then $x + 12,600 =$ B's capital;

and $x + 3,000 =$ C's capital;

Let y = the number of per cent of A's capital;
$y + 1$ = the number of per cent of B's capital;
$y + 2$ = the number of per cent of C's capital.

But the interest of a capital being found by multiplying the capital by the rate of per cent, and dividing by 100, we have

$$\frac{xy}{100} = \text{A's interest};$$

$$\frac{(x + 12{,}600) \times (y + 1)}{100} = \text{B's interest};$$

and $$\frac{(x + 3{,}000) \times (y + 2)}{100} = \text{C's interest}.$$

Now since the excess of B's interest over A, and that of C's over A is known, it will not be difficult to state the two equations.

29) A company expended a certain sum at an inn, which is to be divided equally among them. Had there been 5 persons more, and had each spent $12\frac{1}{2}$ cents more, then the whole bill would have been \$10,$17\frac{1}{2}$; but had there been 3 persons less, and had each expended 5 cents less, the bill would have been \$8,25. How many were there in the company? And how much did they spend apiece?

Remark. Let x = number of persons,
y = number of cents they spent apiece,
then $(x + 5) \times (y + 12\frac{1}{2}) = 1{,}017\frac{1}{2}$ cents.
In a similar manner you may obtain the 2d equation.

30) A work is to be printed, so that each page may contain a certain number of lines, and each line a certain number of letters. If each page was to contain 3 lines more, and each line 4 letters more, then there would be 224 letters more on each page than there are now on it; but if there were 2 lines less in a page and 3 letters less

in each line, then each page would contain 145 letters less than now. How many lines are there on each page? And how many letters are there in each line?

Remark. Let $x =$ number of lines,
$y =$ number of letters in each line,
then $x\,y =$ number of letters on a page,
and $(x + 3) \times (y + 4) = x\,y + 224$;
now find the 2d equation and the rest.

31) Not long ago, says a certain person, the barrel of wheat was by $1, and the barrel of rye by 75 cents cheaper than they are now; then the price of the wheat was double that of the rye; their present prices are as 20 : 11. What is the present price of each article?

Remark. Let $x =$ the present price of the wheat,
$y =$ the present price of the rye.
Then $x - 100$ was double of $y - 75$, or

$$x - 100 = 2\,y - 150;$$

now find the rest.

32) A person had two barrels, and a certain quantity of wine in each. In order to have an *equal* quantity in each, he poured out as much of the first cask into the 2d as the 2d cask already contained; then again, he poured out as much of the 2d into the first as the first then contained; and lastly, he poured out as much from the first into the 2d as there was then still remaining in the 2d. He had then 16 gallons of wine in each cask. How many gallons did each contain at first?

Remark. Let $x =$ the number of gallons of the 1st,
$y =$ the number of gallons of the 2d.

After the first pouring out, the contents will be as follows:

$$x - y = \text{the 1st},$$
$$2\,y = \text{the 2d}.$$

After the 2d pouring out, the contents are,

$$2x - 2y = \text{ in the 1st,}$$

and $2y - (2x - 2y) = 2y - 2x + 2y = 4y - 2x$ in the 2d.

After the third pouring out, &c.

The contents of each must finally be 16 gallons.

33) A wine merchant had two kinds of wine. If he mixes 3 gallons of the worst with 5 of the best, the mixture is worth $1 per gallon; but if he mixes $3\frac{1}{4}$ gallons of the worst with $8\frac{3}{4}$ of the best, then the mixture is worth $1,03$\frac{1}{3}$ per gallon. What does each wine cost per gallon?

34) Find a fraction such, that if 1 be subtracted from the numerator, and 2 from the denominator, it may become equal to 1; but if 1 be *added* to the denominator, and 2 to the numerator, it may also be equal to 1. What fraction is it?

35) 37 lbs. of tin lose 5 lbs of their weight, when plunged into water,* and 23 lbs. of lead lose 2 lbs. in water. A composition of tin and lead weighing 120 lbs. loses 14 lbs. in water. How much does this composition contain of each metal?

Remark. Let $x =$ the number of lbs. of tin in the composition,

$y =$ the number of lbs. of lead in the composition.

If 37 lbs. of tin lose in water 5 lbs.,

1 lb. of tin loses $\frac{5}{37}$.

Again; if 23 lbs. of lead lose in water 2 lbs.,

1 lb. of lead loses $\frac{2}{23}$,

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* How this is to be understood, and how upon it the theory of specific gravity is founded is explained in Grund's Natural Philosophy, under the head of specific gravity.

consequently x lbs. of tin lose $\frac{5}{37}\,x$,
and y lbs. of lead lose $\frac{2}{23}\,y$.

Now that the number of lbs. in the composition, and also that of its loss in water is known, it will not be difficult to state two equations.

36) 21 lbs. of silver lose 2 lbs. in water, and 9 lbs. of copper lose 1 lb. in water. Now if a composition of silver and copper weighing 148 lbs. loses $14\frac{2}{3}$ lbs. in water, how many lbs. does it contain of each metal ?

(This problem is entirely similar to the preceding one.)

37) According to Vitruvius, the crown of king Hiero of Syracuse, weighed 20 lbs. and lost $1\frac{1}{4}$ lbs. in water. Supposing it consisted of gold and silver only, and that 19.64 lbs. of gold lose 1 lb. in water; but 10.50 lbs. of silver only 1 lb., how much gold, and how much silver did this crown contain ?

38) It is required to find two numbers, whose difference, sum, and product are to one another as 2 to 3 to 5, that is, whose difference is $\frac{2}{3}$ of their sum, and whose sum is $\frac{3}{5}$ of the product. What numbers are they ?

Remark. Let x and y be the two numbers,
then $x - y = \frac{2}{3}$ of $(x + y) = \frac{2}{3}\,x + \frac{2}{3}\,y$
and $x + y = \frac{3}{5}$ of $x\,y = \frac{3}{5}\,x\,y$.

Determine x from the first equation, and substitute its value in the 2d, by which means the 2d equation will become divisible by y.

39) Find two numbers, the sum of which is 13, and the difference of their squares 39. What numbers are they ?

Remark. Let x and y be the two numbers.
then $x + y = 13 \ldots.(A)$
and $x^2 - y^2 = 39 \ldots.(B)$

Divide the lower equation (B) by the first equation (A,) viz. $x^2 - y^2$ by $x + y$; and 39 by 13, by which means you will get an equation giving you the *difference* between x and y only; but the sum (A) and difference between two numbers being known, the numbers themselves are easily found.

(See problem 1st, Section IX.)

40) The sum of two numbers is 9; the quotient obtained by dividing the larger by the smaller is 2. What are the two numbers.

Remark. This problem may as well be solved by one unknown quantity only; for if the quotient of $\frac{x}{y} = 2$, x must be equal to $2\,y$; that is, one number must be double of the other.

41) The difference of two numbers is 12, and the one is 4 times contained in the other. What are the two numbers?

42) A young person was asked his own, his father's, and his grandfather's ages. He answered, my age and my father's together, amount to 54; my father's and my grandfather's to 109; and mine and my grandfather's to 85. How old was each?

43) How old are you? asked a son of his father. 7 years ago, answered he, I was 3 times as old as you; 7 years hence I shall only be twice as old. What is the age of each?

44) A, B, and C, owe together $2,190, and none of them can alone pay this sum; but when they unite, it can be done in the following way: 1st, by B's putting $\frac{3}{7}$ of his property to A's; 2d, by C's putting $\frac{5}{8}$ of his property

to B's, and 3dly, by A's adding $\frac{2}{3}$ of his property to C's. How much did each possess ?

45) A and B possess together, only $\frac{2}{3}$ of the property of C ; B and C have together 6 times as much as A ; if B were $680 richer than he actually is, then he would have as much as A and C together. How much has each ?

Remark. Let the three equations

$$x + y = \tfrac{2}{3}z$$
$$y + z = 6x$$
and
$$x + z = y + 680;$$

be reduced to this form,

$$\begin{aligned} x + y - \tfrac{2}{3}z &= 0 \\ -6x + y + z &= 0 \\ x - y + z &= 680. \end{aligned}$$

Adding the last of these equations first to the 2d, and then again to the first, you will get rid of the y ; the rest of the solution is self-evident.

46) I have three purses, A, B, and C, lying before me. If I take $20 out of the first and put them into the 2d, then the 2d contains 4 times as much as the first. If, on the other hand, I take $60 out of the 2d and put them into the 3d, then this contains $1\frac{3}{4}$ as much as there remains in the 2d. But if I take $40 out of the 3d and put them into the first, then the 3d contains $2\frac{7}{8}$ as much as the first. What is the contents of each purse ?

Remark. Let x, y, z, represent the different contents of the three purses,

then $x - 20$ = what remains in the first when $20 are put into the 2d.

$y + 20$ = the contents of the 2d after receiving $20 from the first, consequently, agreeable to condition 1st,

$$y + 20 = 4x - 80, \text{ (1st equation)}$$

$$\text{or } -4x + y = -100.$$

In a similar manner may the 2d and 3d equation be obtained.

47) A, B, and C, compare their fortunes. A says to B, give me $700 of your money, and I shall have twice as much as you retain. B says to C, give me $1,400 and I shall have 3 times as much as you have remaining. C says to A, give me $420 and then I shall have 5 times as much as you retain. How much has each?

48) Find three numbers from the following properties: if 4 are subtracted from the first, and 4 added to the 2d, then the remainder is $\frac{1}{2}$ of the sum. If 10 be subtracted from the 2d, and as many added to the 3d, then the remainder is $\frac{3}{10}$ of the sum; but if 5 be subtracted from the first, and this number be added to the 3d, then the remainder is $\frac{3}{11}$ of the sum. What three numbers are these?

Remark. Let x, y, z, be the three numbers. The 2d equation will be

$$y - 10 = \tfrac{3}{10}(z + 10);$$

which, when multiplied by 10, may be transformed into

$$10y - 100 = 3(z + 10)$$

$$\text{or } 10y - 100 = 3z + 30$$

$$\text{or lastly } 10y - 3z = 130.$$

In a similar manner may the first and 3d equation be arranged.

49) A, B, and C, possess $1,820. If B gives A $200 of his money, then A will have $160 more than B; but if B receives $70 from C, then both (B and C) will have the same sum. How much has each?

50) Three persons, A, B, and C, jointly spent a certain

sum; but none of them is alone able to pay it. A therefore says to B, give me the 4th part of your money, and then I can pay it alone; B says to C give me the eighth part of your money, then I also can pay it; then says C to A, I shall also be able to pay it if I receive half your money, although I, at present, possess only \$4. How much money did they spend? And how much had A and B?

Remark. Let $x =$ A's money,
$y =$ B's money,
$s =$ the whole sum they owed,
then $x + y + 4 = s$; because the 3d possesses only \$4,
or $x + y - s = -4$,
we have farther, $x + \frac{1}{4} y = s$,
$y + \frac{1}{8} \quad = s$,
$\frac{1}{2} x \times + 4 \quad = s$, whence, &c.

51) A person has three pieces of silver of different alloy, viz. 15, 10, and 9 parts out of 16 pure. If he melts the 15 pure with the 10 pure, then the mixture contains 12 parts pure out of the 16, that is, $\frac{12}{16}$ pure. The silver arising from the mixture of the 15 and 9 parts pure, is of the same purity, ($\frac{12}{16}$). All three pieces together weigh 144 ounces. How much does each piece weigh by itself?

Remark. Let x, y, z, respectively, represent the number of ounces each weighs. Then, according to the first condition, we have

$$\frac{15}{16} x + \frac{10}{16} y = \frac{12}{16} \text{ of } (x + y)$$

or $\frac{15}{16} x + \frac{10}{16} y = \frac{12}{16} x + \frac{12}{16} y$,
or also $\frac{3}{16} x = \frac{2}{16} y$, or finally, $3x = 2y$, whence $y = \frac{3}{2} x$.

In a similar manner we obtain from the 2d condition,

$$\frac{15}{16} x + \frac{9}{16} z = \frac{12}{16} \text{ of } (x + z),$$

or $\frac{15}{16} x + \frac{9}{16} z = \frac{12}{16} x + \frac{12}{16} z$, whence $\frac{3}{16} x = \frac{3}{16} z$,
or $x = z$.

Now since by 3d condition, $x + y + z = 144$,
and $y = \frac{3}{2} x$, and $z = x$,
we shall have $x + \frac{3}{2} x + x = 144$,
whence, &c.

52) A person has three magazines, each of which contains three kinds of grain, viz. wheat, rye, and barley. The first magazine contains 192 barrels of wheat, 63 barrels of rye, and 120 barrels of barley; the 2d, 72 barrels of wheat, 210 barrels of rye, and 168 barrels of barley; the third 144 barrels of wheat, 162 barrels of rye, and 312 barrels of barley. The value of the first magazine is $2,202; the value of the 2d, $2,436; and the value of the third, $3,225. What is the value of a barrel of each grain?

53) A, B, and C, purchase coffee, sugar, and tea at the same prices. A pays $11,62½ for 7½ lbs. of coffee, 3 lbs. of sugar, and 2½ lbs. of tea; B pays $16,25 for 9 lbs. of coffee, 7 lbs. sugar, and 3 lbs. of tea; C pays $12,25 for 2 lbs. of coffee, 5½ lbs. of sugar, and 4 lbs. of tea. What is the price of each article per lb.?

54) Three soldiers, A, B, and C, in a battle made $96 booty, which they wish to share equally. In order to do this, A, who made most, gives B and C as much as they already had; after this B gave to A and C as much as they already had; and finally C to A and B as much as they then had. If then by these means the intended equal division is effected, how much did each soldier make?

Solution. Let x, y, z, represent the sums the three soldiers had at first.

According to the first condition we have

$$x + y + z = 96.$$

According to 2d condition, the first soldier has after the first distribution

$$x - y - z,$$

and the others $2y$, $2z$ respectively.

After the 2d distribution,

the 2d had $2y - (x - y - z) - 2z = 3y - x - z$;

the 1st had $2 \times (x - y - z) = 2x - 2y - 2z$;
the 3d had $2 \times 2z = 4z$.

After the third distribution
the third had $4z-(3y-x-z)-(2x-2y-2z)$ [*]
$$4z-3y+x+z-2x+2y+2z=7z-y-x;$$
the 2d had twice $(3y-x-z)=6y-2x-2z$; and
the 1st had twice $(2x-2y-2z)=4x-4y-4z$.

But as the three last shares are all equal to one another, we have the following two equations:

$$7z - y - x = 6y - 2x - 2z$$
$$4x - 4y - 4z = 6y - 2x - 2z;$$

and lastly from 1st condition, $x + y + z = 96$, which three equations, when properly arranged, give

$$\begin{aligned} 9z - 7y + x &= 0 \quad (A) \\ -2z - 10y + 6x &= 0 \quad (B) \\ z + y + x &= 96 \quad (C) \end{aligned}$$

Subtracting (A) from (C) we obtain
$$-8z + 8y = 96 \quad (a)$$
Multiplying (C) by 6 and subtracting (B) from it we have
$$8z + 16y = 576 \quad (b)$$
and adding the two last equations (a) (b) together, we have at once,
$$24y = 672,$$
whence the value of z and x are easily found.

55) There are three numbers whose sum is 83. If from the first and from the 2d (from each) 7 be subtracted, the remainder of the 2d is $\frac{3}{5}$ of the remainder of the first; but if from the 2d and 3d (from each) 3 be subtracted, then the remainder of the 3d is $\frac{9}{11}$ of the remainder of the second. What three numbers are they?

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[*] Compare to this the problems in Subtraction of compound quantities, Sect. I.

Remark. Let the three numbers be represented by x, y, z, respectively.

Then according to the 1st condition,

$$x + y + z = 83.$$

According to the 2d condition,

$$y - 7 = \tfrac{3}{5} \text{ of } x - 7,$$

$$\text{or } y - 7 = \tfrac{3}{5} x - \tfrac{21}{5}, \text{ \&c.}$$

56) In the three drawers of my desk I have a sum of $162. In order that there may be the same sum in each, I take out as much of the first as is necessary, in order to put into the other two half of what each already contained. I then take out of the 2d, and afterwards out of the third drawer, each time as much as is necessary to put into the other two, half of what they then contained. Now if at the end of the third distribution, there remained actually the same sum in each, how much did each drawer contain at first?

(See the solution of problem 54.)

57) Required to find three numbers of the following properties: if 6 be added to the first and to the 2d, then the first sum is $\frac{2}{3}$ of the 2d sum; when 5 are added to the first and to the 3d, the 2d sum is $\frac{7}{11}$ of the 3d; but if 36 be *subtracted* from the 2d and from the 3d, then the remainder of the 2d is $\frac{6}{7}$ of the remainder of the 3d. What numbers are these?

Remark. The solution of this problem is analogous to that of problem 55.

SECTION XI.

PROBLEMS WHICH LEAD TO QUADRATIC EQUATIONS.

A. Problems which lead to pure Quadratics.

1) What number is it whose square is 144?

2) What number is it, whose half raised to the square gives 49?

3) What number is it, whose half multiplied by its fourth, gives 18?

4) What number is it, whose half multiplied by its third part gives 864?

5) What number is it, whose 5th and 6th part multiplied together gives 480?

6) What number is it, whose 7th and 8th part multiplied together, and the product divided by 3, gives the quotient 298$\frac{2}{3}$?

7) It is required to find a number such, that if it be added to 94, and again subtracted from 94, the product of the sum by the difference thus obtained, shall be 8,512. What number is it?

Remark. If $x =$ the number,
then $x + 94$ is the sum which must be multiplied by $x - 94$ (the difference) in order to produce 8,512.

8) What two numbers multiplied together give 750, and when divided by each other, give the quotient 3$\frac{1}{3}$?

Remark. If one number is represented by x, then on account of condition 2d, the other is $3\frac{1}{3}x$.

9) Find two numbers, the sum of whose squares $= 13,001$, and the difference of their squares $= 1,449$.

Remark. The pupil ought to observe that the sum of the squares added to the difference gives twice the square of the larger.

10) Some persons asked the landlord the amount of their bill. He answered, if each of you pay as many shillings as there are persons, I am satisfied. They then paid the landlord \$24, (6 shillings $=$ \$1.)

11) A gentleman bought a number of horses; each costs 4 times as many dollars as there are horses. How many horses did he buy?

12) If I multiply the tenth part of my fortune by itself, and obtain \$419,904, what is my fortune?

13) A gentleman has three sorts of merchandise. The lbs. of each sort costs as many dollars as it contains lbs. The 2d sort contains twice as many lbs. as the first, and the third sort contains 3 times as many pounds as the 2d. How many lbs. were there of each sort?

14) A certain number of merchants have put out a capital at interest; each has put out 10 times as many dollars as there are merchants, and gains as many per cent as there are merchants. Now if the 100th part of the whole gain is multiplied by the number $2\frac{2}{8}$, the product is the number of merchants. How many merchants were there?

Remark. If $x =$ number of merchants,
then $10x =$ the capital of each,
$10x^2 =$ the whole capital put out,

$\frac{x}{100}$ of $10x^2 = \frac{x^3}{10}$ = the interest or gain of the whole, &c.

(The final equation will contain x in both terms, and may therefore be divided by x.)

15) A certain capital is put out at 4 per cent. If you multiply the number of dollars in the capital by the number of dollars in the interest for 5 months, you obtain \$117,041$\frac{2}{3}$. What is the capital?

Remark. Let x = the number of dollars in the capital,
then $\frac{4}{100}x = \frac{1}{25}x$ = number of dollars in the interest of 1 year or 12 months,
and $\frac{5}{12}$ of $\frac{1}{25}x = \frac{1}{60}x$ = number of dollars in the interest of 5 months.

The rest of the solution involves no further difficulty.

16) A person has three kinds of goods, which together cost \$230$\frac{5}{24}$. The pound of each article costs as many three pences ($\frac{1}{24}$ of a dollar) as there are lbs. of that article. But he has $\frac{1}{3}$ more lbs. of the 2d kind than he has of the first, and $3\frac{1}{2}$ times as many of the third as he has of the 2d. How many lbs. has he of each article?

Remark. If x = number of lbs. of the first kind,
$\frac{4}{3}x$ = number of lbs. of the 2d kind,
and $3\frac{1}{2} \times \frac{4}{3}x$ = number of lbs. of the 3d kind.

(The rest of the solution is analogous to that of problem 14.)

17) A person has a small quantity of a certain merchandise on hand. On being asked how many lbs. there were of it, he answered, if I sold the lb. for $2\frac{3}{5}$ times as many dollars as there are lbs. I should get exactly as much above \$125, as I shall get less than \$125 if I sell the lbs. for half as many dollars as there are lbs. How many lbs. had he?

Remark. Let $x =$ number of lbs.
$2\frac{3}{5}x = \frac{13}{5}x$ the number of dollars he would sell a lb. in the first case,
$\frac{13}{5}x^2 =$ the price of the whole merchandise in this case,
and $\frac{13}{5}x^2 - 125 =$ to what he would then get *above* \$125.

Again, $\frac{x}{2} =$ number of dollars he would sell a lb. in the 2d case.

The rest of the solution is sufficiently plain.

18) I have a certain number in my thoughts; this I multiply by $2\frac{1}{3}$, add 7 to the product, and multiply this sum by 8 times the number. I then divide by 14, and subtract from the quotient 4 times the number, and obtain 2,352. What number is it?

Remark. Although this problem appears to be complicated, it is yet one of the simplest. If the pupil denotes the unknown number by x, and performs correctly all the operations indicated in the enunciation, the final equation is a simple quadratic.

B. Problems leading to Mixed Quadratic Equations.

1) There are two numbers, one of which is greater than the other by 8, and whose product is 240. What numbers are they?

2) Some persons had drank 12 bottles of wine, at \$1 a bottle. The landlord told them that he should be satisfied if each of them would pay him 59 cents more than there are persons. How many persons were there?

3) A field is 6 roods longer than it is broad; its whole

contents is 91 square roods. How long and how broad is that field?

Remark. The pupil ought to recollect that the square contents of a rectangular field is the product of its length by its breadth.

4) It is required to find a number, whose square is greater than the number itself by 306?

5) It is required to find a number such, that if its 3d part be multiplied by its 4th, and to the product 5 times the number added, the sum exceeds the number 200 by as much as the number sought is less than 280?

Remark. Let $x =$ that number,

then $\frac{x}{3} \times \frac{x}{4} = \frac{x^2}{12} =$ its third multiplied by its fourth,

$\frac{x^2}{12} + 5x - 200 =$ what the product, plus 5 times the number exceed 200,

$280 - x =$ what the number is less than 280.

The rest of the solution is self-evident.

6) A gentleman stopped with his family at a tavern, and paid a certain sum for each day. He remained there 6 days longer than he pays dollars for each day. At last his bill amounts to $135. How many days was he at the tavern? And how many dollars did he pay for each day?

Remark. Let $x =$ the number of dollars he pays each day,

then $x + 6 =$ number of days he stopped,

and $x + 6$ multiplied by $x =$ what he paid for the whole time.

The rest of the solution is sufficiently easy.

7) A person being asked his age, answered, my mother was 20 years old when I was born, and her age multiplied by mine, exceeds our united ages by 2,500. What was his age?

Remark. If $x =$ his age,
then $x + 20 =$ his mother's age,
and x multiplied by $(x + 20)$, less $(2x + 20)$ must equal 2,500.

8) A farmer being asked how many oxen he had in his stable, answered, if I multiply their number by itself, then the product is by 32 greater than the result which I obtain if I multiply their number by 35, and add to the product the number itself, together with 800. How many oxen were there?

9) A man bought a horse for a certain number of dollars, and sold it again for $119, by which means he gains as many per cent as the horse cost him dollars. What did the horse cost him?

Remark. Let $x =$ what the horse cost him,
then the gain is $\frac{x}{100}$ of x, or $\frac{x^2}{100}$.

10) Determine the fortunes of three persons, A, B, and C, from the following properties: for every $5 which A possesses, B has $9 and C $10. Farther, if we multiply A's money by B's, and B's money by C's, and add both products to the united fortunes of all three, we shall get 8,832. How much had each?

Remark. Let $x =$ C's fortune,
then $\frac{9}{10}x =$ B's,
and $\frac{1}{2}x =$ A's.
The rest of the solution is evident from the conditions.

11) A person bought some pieces of cloth at equal prices for $60. Had he got three pieces more for the same sum, each piece would have cost him $1 less. How many pieces did he buy?

Remark. Let $x =$ the number of pieces, then $\frac{60}{x} =$ the price of 1 piece.

But if he had got 3 pieces more for the same sum, then

$\frac{60}{x+3} =$ the price of 1 piece.

12) A charitable person divides the sum of $36, in equal shares among the poor of a small town; but as 6 of those whom he thought of relieving stood no longer in need of assistance, each of the remaining paupers had for his share sixpence ($\frac{1}{12}$ dollar) more than he otherwise would have had. How many paupers were there at first?

(This problem is entirely similar to the preceding one.)

13) In a town there are a certain number of houses. If 55 be added to their number, and 45 subtracted from it, and the sum and difference thus obtained are multiplied by each other, you obtain 900,000. How many houses are there in that town?

Remark. If $x =$ the number of houses, the $x + 55$ is the sum which is to be multiplied by the difference $x - 45$.

14) Three journeymen were working each as many days as he had shillings for his daily wages; but B has 1 shilling a day more than A, and C one shilling more than B. They received together $40 and 5 shillings. How many days did each of them work? And how much did each of them earn?

15) Each of two captains ordered $1,200 booty to be divided *equally* among his soldiers. The second captain had 40 men less in his company than the first, on which account each of his men received $5 more than one of the other company. How many soldiers had each captain? And how much does each soldier receive?

Remark. Let x = the number of the 2d company,
then $x + 40$ = the number of the 1st company,
$\frac{1200}{x}$ = the share of 1 man in the 2d company,
and $\frac{1200}{x + 40}$ = the share of 1 man in the 1st company.

16) A farmer has a field of 1,680 square roods. The length of this field is by 4 greater than 4 times its breadth. How long and how broad is this field?

(See remark to problem 3, page 173.)

17) I have a number of dollars about me. If I multiply their number less 5, by their number less 6, then I obtain 3 times their number. How many dollars have I?

18) A person bought as many pounds of a certain commodity as he pays pence per lb.; but for paying cash he deducts a penny per lb., paying in the whole only £4 19*s.* 3*d.* How many lbs. did he buy?

19) A gentleman being asked his age, answered, my brother is 1 year older than I, and his age multiplied by mine produces the number 1,260. What is the age of each?

20) 20 persons, men and women, spent together $48 at an inn: the men $24, and the women the same sum. Now on inspecting the bill, it is found that the men have to pay one dollar each more than the women. How many men and women were there in the company?

(This problem is similar to problem 15.)

If x = number of men,
$20 - x$ = number of women,
$\frac{24}{x}$ = what 1 man spends, &c.

21) A person bought a horse, and paid a certain sum for him. He afterwards sells him again for \$144; by which means he gains exactly as much per cent as the horse cost him. How much did the horse cost?

(Compare this to problem 9, page 174.)

22) There are two numbers, one of which is by 7 greater than the other. If the number 100 be divided by each of these numbers, and the quotients are added, $43\frac{1}{3}$ are obtained. What two numbers are these?

23) A merchant sends for a piece of stuff, and pays a certain sum for it, besides 4 per cent for carriage. He sells it again for \$390, and gains by the bargain as much per cent as the $12\frac{1}{2}$th part of the purchase money amounted to. What did he buy it for?

Solution. Let $x =$ number of dollars he gave for it,

then $\frac{4}{100}x = \frac{1}{25}x$ what he paid for carriage,

and $x + \frac{1}{25}x = \frac{26}{25}x =$ what the whole stands him in for,

$\frac{x}{12} =$ the 12th part of the purchase money;

consequently, also the number of per cent he gains;

therefore, $\frac{x}{1200}$ of $\frac{26}{25}x =$ the whole gain, which together with the cost $(\frac{26}{25}x)$ must amount to \$390.

The rest may easily be found by the pupil.

24) A lb of a certain commodity is sold with as many per cent profit as the prime cost of it amounted to in dollars. It is sold for \$24 a lb. What was the prime cost of this commodity?

(This problem is easier, but similar to the preceding one.)

25) Two travellers, A and B, set out at the same time from two different places, *c*, *d*. A travels from *c* to *d*, and

B from d to c. On the way they met, and it then appeared that A had already gone 30 miles more than B, and according to the rate at which they travel, A calculates to reach the place d in 4 days, and that B can arrive at the place c in 9 days. What is the distance (how many miles are there) between c and d?

Solution. Let $x =$ the number of miles B travelled,
then $x + 30 =$ the number of miles A travelled; therefore B will have left $x + 30$ miles to make, which is the distance A has come, in order to reach c; and A will have x miles to make, or the distance which B had come when they met, in order to reach d.

But as B calculates to be in c in 9 days, he must travel at the rate of

$$\tfrac{1}{9} \text{ of } x + 30 = \frac{x + 30}{9} \text{ in 1 day,}$$

and for a similar reason must A travel at the rate of

$$\frac{x}{4} \text{ in 1 day.}$$

Now as B has come x miles, and travelled $\frac{x + 30}{9}$ miles in a day, we obtain the number of days he was out on his journey *before* he met A, by dividing the distance he has come by the number of miles he travels in 1 day. By this means we obtain

$$x : \frac{x + 30}{9} = \frac{9x}{x + 30}$$

for the number of days B had been on the road; and in a similar manner,

$$(x + 30) : \frac{x}{4} = \frac{4x + 120}{x}$$

for the number of days A had been on the road.

Now in the problem it is stated that they set out at the *same time;* they must consequently have been the same number of days on the road when they met; whence it is easy to form an equation; and if x is once determined, the rest follows of course.

26) What two numbers are they, whose sum is 41, and the sum of whose squares is 901?

Remark. Let x and y respectively represent the two numbers.

Then $x + y = 41$ (A)
and $x^2 + y^2 = 91$ (B)

Raise both members of the first equation (A) to the square, and you will obtain

$$\begin{array}{rl} & x^2 + 2xy + y^2 = 1{,}681, \\ \text{and as} & \underline{x^2 \qquad\quad + y^2 = \quad 91.} \\ & \qquad 2xy \qquad = 1{,}590, \\ & \text{and} \quad x = \frac{1{,}590}{2y} = \frac{795}{y}. \end{array}$$

This value of x substituted in (A) will give y, whence the rest follows of course.

27) What two numbers are they, whose difference is 8, and the sum of whose squares is 544?

The two equations are $x - y = 8$ (A)
$x^2 + y^2 = 544$ (B)

Proceed with them as in the preceding problem.

Remark. Let the pupil recollect that the square of $-y$ is $+y^2$.

28) Find two numbers whose product is 255, and the sum of whose squares is 514.

Remark. Let x, y, respectively represent these numbers.

Then $x \times y = 255$ (A), whence $x = \frac{255}{y}$
and $x^2 + y^2 = 514$ (B).

Now if the pupil were to proceed as usual, he would obtain an equation of the 3d degree; but if he pay proper attention to the nature of the two equations before him, he will easily perceive that by adding double of the 1st equation to the 2d (B), the 2d equation becomes a perfect square, viz.

$2\,x\,y = 510$, which added to (B)

gives $x^2 + 2\,x\,y + y^2 = 514 + 510$,

or $x^2 + 2\,x\,y + y^2 = 1{,}024$.

Now by extracting the square root of both numbers, he will obtain a simple equation in x and y; which together with the equation (A) is sufficient to determine x and y.

29) Divide the number 16 into two such parts, that the product of the two parts added to their squares, may be $= 208$.

Remark. If one number is represented by x, the other is $16 - x$.

(In taking the square of $16 - x$ the pupil must take care not to omit the double product of 16 by $-x$.

30) What number is it, which added to its square root gives 1332?

Remark. If $x =$ the number,

then $\sqrt{x} =$ the square root of it,

and $x + \sqrt{x} = 1332$.

Now to get rid of the term $\sqrt{x}$, we must raise both members of the equation to the square. But if we do this with the equation as it now stands, we shall in taking the double product of x by $\sqrt{x}$, obtain a new term, involving $\sqrt{x}$. For this reason we must give our equation the form

$$\sqrt{x} = 1332 - x,$$

and now by raising both numbers to the square,

$$x = 1{,}774{,}224 - 2{,}664\,x + x^2,$$

or, by bringing all the terms which involve x on the left side of the equation,

$$-x^2 + 2{,}665\,x = 1{,}774{,}224,$$

and the changing all the signs, (that is, by multiplying the whole equation by -1),

$$x^2 - 2{,}665\,x = -1{,}774{,}224, \text{ whence } x^2 = \&c.$$

Of the two signs, which in the answer precede the radical sign ($\sqrt{}$), only the sign — can be used in this problem.

31) What number exceeds its square root by $48\frac{3}{4}$?

32) Two retailers, A and B, jointly invested $500 in business, to which each contributed a certain sum. A let his money remain 5 months, B only 2 months, and each received back $450 capital and profit. How much did each advance?

Solution. Since both together invested $500, and each received back $450, they received together $900; the whole profit therefore was

$$900 - 500 = 400 \text{ dollars.}$$

Now if $x =$ the number of dollars the first contributed,

then $500 - x =$ the number of dollars the second contributed.

But in compound fellowship, the whole capital, multiplied by its time, is to each partner's capital multiplied by its time, as the whole gain is to each partner's gain.

Now we have

$$\begin{aligned} 5\,.\,x &= \text{A's capital multiplied by its time,} \\ \underline{1{,}000 - 2\,x} &= \text{B's capital multiplied by its time,} \\ \text{therefore } 1{,}000 + 3\,x &= \text{the whole capital multiplied by its time;} \end{aligned}$$

consequently we have the proportion,

$1{,}000 + 3\,x : 5\,x :: 400 :$ A's share of the gain, whence $\frac{2{,}000\,x}{1{,}000 + 3\,x} = \text{A's gain.}$

(In a similar manner we might have obtained

$$1{,}000 + 3\,x : 1{,}000 - 2\,x = 400 : \text{B's gain};$$

whence $\frac{400{,}000 - 800\,x}{1{,}000 + 3\,x} = \text{B's gain.}$

But for the purpose of our solution A's gain is sufficient.)

Now as A received for capital and gain $450, we must have the equation

$$\frac{2{,}000\,x}{1{,}000 + 3\,x} + x = 450.$$

Multiplying the whole equation by $1{,}000 + 3\,x$, we obtain

$$2{,}000\,x + 1{,}000\,x + 3\,x^2 = 450{,}000 + 1{,}350\,x,$$

or $3\,x^2 + 1{,}650\,x = 450{,}000,$

and $x^2 + 550\,x = 150{,}000;$

whence $x = \text{\&c.}$

The same problem might have been solved also by the following proportion: A's capital multiplied by its gain, is to B's capital by its gain, as A's gain is to B's gain; for if each partner's gain is in proportion to the whole gain, their gains must be in proportion to each other, which may be expressed thus:

$$5\,x : 1{,}000 - 2\,x :: 450 - x : 450 - (500 - x);$$

or $5\,x : 1{,}000 - 2\,x :: 450 - x : -50 + x;$

(because $450 - x$ is A's gain,

and $450 - (500 - x) = -50 + x$ is B's gain,)

and because the product of the means is equal to the product of the extremes,

$$450{,}000 - 900\,x - 1{,}000\,x + 2\,x^2 = 5\,x^2 - 250\,x.$$

This equation, when reduced, may be written in this form,

$$-3\,x^2 - 1{,}650\,x = -450{,}000,$$

$$3\,x^2 - 1{,}650\,x = 450{,}000,$$

and lastly $x^2 + 550\,x = 150{,}000;$

which is exactly the same equation as the one we obtained above.

(Let the pupil now try to solve the following problem by himself.)

33) Two persons, A and B, jointly invested $2,000 in business. A let his money remain 17 months, and received back in capital and profit $1,710; the other allowed his money to remain 12 months, and received in capital and gain $1,040. What was each partner's stock in trade?

34) Find two numbers whose difference added to the difference of their squares makes 150, and whose sum added to the sum of their squares makes 330. What two numbers are these?

Remark. If x and y represent these two numbers respectively; then we have the two equations

$$x^2 - y^2 + x - y = 150$$

and

$$x^2 + y^2 + x + y = 330$$

and by adding the two equations,

$$2\,x^2 + 2\,x = 480, \text{ whence, \&c.}$$

By subtracting the upper equation from the lower, we should have obtained

$$2\,y^2 + 2\,y = 180, \text{ whence, \&c.}$$

35) It is required to find a number, consisting of three digits, such, that the sum of the squares of the digits, without considering their position, may be $= 104$; but the square of the middle digit exceeds twice the product of the other two by 4; further, if 594 be subtracted from the number sought, the three digits become inverted. What number is it?

Remark. Let x represent the first digit beginning on the right,

y the second,
z the third,

then the number is $100z + 10y + x$,
(because in the third place are the hundreds, in the 2d the tens, and in the 1st the units.)

The three conditions of our problem may now be verified in the following manner:
according to the 1st condition, we have

$$x^2 + y^2 + z^2 = 104; \quad . \quad (A)$$

according to the 2d condition,

$$y^2 = 2xz + 4; \quad . \quad . \quad (B)$$

and according to condition 3d,

$$100z + 10y + x - 594 = 100x + 10y + z \quad . \quad (C)$$

(because the digits shall then be inverted.)

This last equation when arranged gives

$$99z - 99x = 594,$$
$$\text{or} \quad 11z - 11x = 66,$$
$$\text{or} \quad z - x = 6,$$
$$\text{whence} \quad z = x + 6 \quad . \quad . \quad (a)$$

This value of z substituted in (B) gives

$$y^2 = 2x(x + 6) + 4;$$
$$\text{or} \quad y^2 = 2x^2 + 12x + 4.$$

Further, since $z = x + 6$ . . . . . (a)

$$z^2 = x^2 + 12x + 36.$$

These two values of y^2 and z^2 substituted in (A) give

$$x^2 + 2x^2 + 12x + 4 + x^2 + 12x + 36 = 104,$$

or $4x^2 + 24x = 64$; whence, &c.

The value of x substituted in (a) determines z and the values of x and z substituted in (B) gives y.

Let the pupil now solve the following problem by himself.

36) It is required to find a number consisting of three digits, by the following conditions: the sum of the squares of the digits, without considering their position, shall be

77; the square of the middle digit shall by 1 exceed the product of the other two; further, if 198 be subtracted from the whole number sought, the three digits become inverted. What number is it?

37) There are three numbers in a geometrical progression. If we add them their sum is 126; but if we multiply them together their product is 13,824. What three numbers are they?

Remark. Let x, y, z, represent these numbers respectively.

Then because they are in a geometrical proportion

$$\frac{x}{y}=\frac{y}{z}, \qquad (A)$$

further, their sum being 126,

$$x+y+z=126, \qquad (B)$$

finally, their product $xyz=13{,}824.$ (C)

From (A) we have $x\,z=y^2$

$$\text{and } x=\frac{y^2}{z} \qquad (a)$$

This value of x substituted in C gives

$$\frac{y^2}{z}\,.\,y\,z=13{,}824\,;$$

$$\text{or} \qquad y^3=13{,}824,$$

$$\text{whence} \qquad y=\sqrt[3]{13,\,824}.$$

The value of y being found, substitute it in (a), and you will have expressed x by means of z only. This new value of x together with that of y substituted in (B) will give the value of z by a simple quadratic; whence the rest follows of course.

APPENDIX.

CONTAINING PROBLEMS IN COMPOUND INTEREST AND ANNUITIES FOR THE USE OF LOGARITHMS.

Formulas.

I. If c denote any capital you please, p the rate of interest (per cent), n the number of years it stands out, then capital and compound interest amount together in n years to $= c\,p^n$, whence the log. of the whole amount A

$$= \log. c + n \log. p.$$

Remark. p in this formula stands for what 100 dollars amount to at the end of one year, divided by 100. Thus if the rate of interest is 4 per cent, then 100 dollars will at the end of 1 year amount to 1.04; wherefore

$$p = \tfrac{104}{100} = 1.04.$$

In like manner if the rate of interest were 5 per cent p would stand for 1.05,

$$\begin{aligned} &\text{if } 6 \text{ per cent, } p = 1.06,\\ &\text{if } 4\tfrac{1}{2} \text{ per cent, } p = 1.04\tfrac{1}{2} = 1.045,\\ &\text{if } 5\tfrac{3}{4} \text{ per cent, } p = 1.05\tfrac{3}{4} = 1.0575,\end{aligned}$$

and so on.

II. If c denote a capital which was put out for n years at the rate of p per cent, and C another capital put out at P per cent; then in order that the latter capital C may become as large as c, we must have the equation

$$N \log P + \log C = n \log p + \log c.\ ^*$$

These two formulas are sufficient to solve any problem in compound interest. If, for instance, the number of years which the 2d capital must stand out to become equal to the first is demanded, then you need only make N the unknown quantity; if instead of N, the amount of the capital C, or the rate of interest is to be found, then determine it from the same equations by taking c or p for the unknown quantities, &c.

APPLICATION OF THESE FORMULAS TO THE SOLUTION OF PROBLEMS.

1) A capital of $1,000 is put out at 4 per cent compound interest. What will it amount to in 10 years?

Remark. The logarithm of the amount

$$\text{is} = \log 1{,}000 + 10 \times \log 1.04.$$

$$\text{Now } \log 1{,}000 = 3,$$

$$\log 1.04 = 0.012837,$$

$$\text{hence } 10 \log 1.04 = 0.128370,$$

$$\text{consequently } \log 1{,}000 + 10 \log 1.04 = 3.128370.$$

Now the number which corresponds to this logarithm is also the number which corresponds to the logarithm of the whole amount of the capital; and therefore the answer.

In a similar manner find the answer to the following questions.

2) A capital of $1,000 is put out at 5 per cent compound interest. What will it amount to in 20 years?

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[*] Supposing the latter capital C were to remain out N years. Then according to Formula I. we have the whole amount of this capital in N years $= C P^N$. And as its amount is equal that of c in n years, which is $= c p^n$, we have the equation $C P^N = c p^n$, and taking the logarithm of both numbers, we have $\log. C + N \log. P = \log. c + n \log p$.

years, if it bear 6 per cent, and the interest be paid *half yearly*?

Remark. If the interest be paid *half* yearly for 10 years, it is the same as if the same capital were out at 3 per cent annually for 20 years. Our formula becomes in this case,

$$\log \text{ of the amount} = \log 10{,}000 + 20 \log 1.03.$$

14) The Boston Provident Institution for Savings declares semi-annually a dividend of 4 per cent,* which, if not taken up, is added to the capital. What will \$100 at this rate of interest amount to in 10 years?

15) What would be the amount of \$85 put into the Boston Savings Bank, at the end of 12 years?

16) To what sum will \$10 put into the Boston Savings Bank increase in 20 years?

17) A gentleman is to receive \$6,000 in 4 years to come, without interest. What is this sum worth, if compound interest be calculated at 4 per cent?

Remark. From formula I, (page 186)

log of amount $= \log c + n \log p,$

we obtain $\log 6{,}000 = \log c + 4 \log 1.04$

(because here the *amount* at the end of the 4th year is known; but the *capital* which in 4 years amounts to that sum, is to be determined.)

From this equation we have

$$\log 6{,}000 - 4 \log 1.04 = \log c;$$

but $\log 6{,}000 - 4 \log 1.04 = 3.846283;$

therefore the number which corresponds to this logarithm in the tables will be the capital sought.

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3) A capital of $5,000 stands out at 4 per cent compound interest. What will it amount to in 10 years?

4) The same capital (see problem 3) stands out 20 years. What will it amount to at the end of that time, supposing the rate of interest to remain the same?

5) What is the amount of the same capital (problem 3) at the same rate of interest, in 40 years?

6) What will $3,200 at 3 per cent compound interest, amount to in 80 years?

7) What will the same capital amount to in 80 years at the rate of 6 per cent?

8) In a certain province there are at present 200,000 inhabitants. If the population increases $\frac{1}{50}$ part (2 per cent) yearly, what will it be in 100 years hence?

9) In a certain province there are at present 500,000 inhabitants. If the population increases annually by $\frac{1}{10}$ (10 per cent) of the population, what will it amount to in 100 years?

10) A gentleman puts out a capital of $20,000 at 5 per cent compound interest. What will it amount to in 12 years?

11) A gentleman puts out $25,000 at $3\frac{1}{2}$ per cent compound interest. What will it amount to in 20 years?

12) What is the amount of a capital of $10,000 at the rate of 6 per cent in 15 years 8 months?

Remark. Calculate the amount of the capital to the end of the 12th year, to which add, 8 months simple interest of the whole amount found.

13) What will a capital of $12,000 amount to in 10

years, if it bear 6 per cent, and the interest be paid *half yearly* ?

Remark. If the interest be paid *half* yearly for 10 years, it is the same as if the same capital were out at 3 per cent annually for 20 years. Our formula becomes in this case,

$$\text{log of the amount} = \log 10{,}000 + 20 \log 1.03.$$

14) The Boston Provident Institution for Savings declares semi-annually a dividend of 4 per cent,* which, if not taken up, is added to the capital. What will $100 at this rate of interest amount to in 10 years?

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Remark. Calculate the amount of the capital to the end of the 12th year, to which add, 8 months simple interest of the whole amount found.

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therefore the number which corresponds to this logarithm in the tables will be the capital sought.

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* The Boston Savings Bank declares yet every 5 years another dividend, in addition to the 4 per cent semi-annually.

18) A gentleman owes me $1,052, which sum he is to pay me 9 years hence; but he wishes to pay me now, calculating 3½ per cent compound interest. How much must I receive?

19) A gentleman's daughter is now 7 years old. He wishes to give her $1,000 when she is 16 years old. How much capital therefore must he now put out for her at 5 per cent compound interest, in order that it may amount to $1,000 when she is 16 years old?

20) A gentleman who expects to live but 10 years longer, wishes to put out a certain capital at the rate of 4 per cent compound interest, so that his wife may at his death inherit $10,000. How much capital must this gentleman put out at interest?

21) A sailor who expects to go to sea but 12 years longer, wishes to put enough money in the Boston Savings Bank to buy at the end of that time, a small farm for $500. How much money must he now, for that purpose, put into the Savings Bank?

Remark. The pupil ought to recollect that the Boston Savings Bank declares a *semi*-annual dividend of 2 per cent. The question therefore is the same as this: What capital will in 24 years amount to $500, if compound interest be calculated at the rate of 2 per cent? (See the remark to problem 13.)

22) What sum of money must be put out at 6 per cent compound interest, in order that it may amount to $25,000 at the end of 12 years?

23) What sum of money must be put out at 5 per cent compound interest, in order that it may amount to $5,000 at the end of 20 years?

24) What is the capital of $50,000 payable in ten years

hence worth now, if compound interest be reckoned at the rate of 6 per cent?

25) What must be the amount of a capital at 4 per cent, that 12 years hence it may be worth $10,000?

26) A person borrows $600 of a usurer, for which he gives him a bill of $800 payable in 3 years. What was the rate of interest paid annually?

Remark. Taking again the formula I,

$$\log \text{amount } A = \log c + n \log p,$$

we have $$\log 800 = \log 600 + 3 \log p;$$
(because the whole amount at the end of three years is the $800, and the capital $600.)

From this equation we obtain

$$\frac{\log 800 - \log 600}{3} = \log p.$$

But $$\frac{\log 800 - \log 600}{3} = 0.04165;$$

which is therefore the logarithm of p; whence p itself $= 1.1006$; that is, the interest is a little over 10 per cent.

(If we had found $p = 1{,}09$, the rate of interest would have been 9 per cent; having found $p = 1{,}10$, the interest is 10 per cent. See the remark to formula I, page 186.)

27) A gentleman owes me $1,052, due in 9 years hence. He wishes to pay this debt by paying me *now* $800. How high does he calculate the compound interest of that money?

28) What is the *present* value of $6,000 payable in 10 years, if compound interest be calculated at the rate of 5 per cent?

29) But what is the *present* value of the same capital

(problem 28) due in 10 years hence, if compound interest be calculated at 6 per cent ?

30) What is the present value of the same capital (problem 29 and 28) due at the same time, if compound interest be calculated at the rate of 6 per cent; and the interest is to be paid *semi-annually ?*

(See the remarks to problems 13 and 21.)

31) What is the present value of a stock of turf, which 20 years hence produces a sum of $500, so that the money laid down for it may bear 4 per cent compound interest ?

32) By how many per cent must a trader annually increase his capital if he has now but $6,000, and wishes to have $10,000 in 4 years hence ?

33) But by how many per cent must he increase the same capital (problem 32) annually, if he wishes to possess 4 years hence $12,000 ?

34) A capital of $800 was so well laid out, that in the space of 6 years it had increased to $3,600. What per cent did this capital gain annually ?

35) The population of the state of Ohio was in 1790 about 3,000 souls ; in 1800 it was 45,365. By how many per cent annually did this population increase ?

36) The population of the same state increased in the years from 1820 to 1830, from 581,434 to 937,637 souls. By how many per cent annually did it increase during that period ?

37) The city of New York had in 1696 only 4,302 inhabitants. In 1830 it contained 203,007. If this increase be averaged in per cents, by how many per cent annually did this population increase during all that time ?

38) The town of Boston had in 1700 but 7000 inhabitants; but in 1830, 61,392. By how many per cent did the population increase annually during these 130 years?

39) The population of the United States, according to the first census, made in 1790, amounted to 3,929,000 souls nearly. In 1830 it was 12,856,000 nearly. What was the rate of increase during that period?

(The fraction of the answer may be omitted.)

40) If the population of the United States continues to increase at the same rate, (problem 39) what will it amount to in 100 years?

41) In how many years will $1,000 amount to $1,500, calculating 5 per cent compound interest?

Remark. Our formula I, (page 186) gives

$$\log \text{amount } A = \log c + n \log p,$$

$$\text{whence } \log 1{,}500 = \log 1{,}000 + n \log 1.05,$$

$$\text{when } n = \frac{\log 1{,}500 - \log 1{,}000}{\log 1.05}$$

Now $\log 1{,}500 - \log 1{,}000 = 0.17609.$
which divided by $\log 1.05 = 0.02119,$
gives the quotient 8.31.

The answer therefore is between 8 and 9 years.

42) How long must $5,000 stand out at 4 per cent interest, in order that it shall have increased to $8,000?

43) Supposing the population of the United States is now (in 1833) 13,500,000 souls. In how many years hence will it be 20,000,000, supposing the rate of increase to be the same as found in problem 39?

44) How long before $100 put in the Boston Savings Bank, will amount to $500; compound interest being paid semi-annually at the rate of two per cent?

(See the remarks to problem 13, page 189.)

45) How long must \$1 stand out, at the rate of 4 per cent compound interest, in order that it may increase to \$2?

Remark. From formula I, (page 186) we obtain again

$$\log \text{amount } a = \log c + n \log p.$$

But here the amount a to which the capital shall increase is $= 2$.

The capital itself is $= 1$;
consequently we have the equation

$$\log 2 = \log 1 + n \log 1.04;$$

but since $\log 1 = 0$, we have also

$$\log 2 = n \log 1.04;$$

whence $$\frac{\log 2}{\log 1.04} = n.$$

46) How long must \$1,000 stand out at the rate of 4 per cent compound interest, in order that it may increase to \$2,000?

47) How many years must a capital stand out at the rate of 5 per cent compound interest, in order to double itself?

Remark. This problem and the preceding are analogous to that of number 45.

48) In how many years will the population of the U. S which is now (in 1833) 13,500,000 souls, double itself, supposing the rate of increase to be the same as in problem 39?

49) A trader increases his stock annually by 18 per cent. In how many years will he have doubled his stock?

50) How long must \$1 stand out at 4 per cent compound interest, before it increases to \$3? Or, which is the same, how long must a capital stand out at 4 per cent compound interest, that it may triple itself?

51) How long must a capital stand out at the rate of 5 per cent compound interest, in order to triple itself? How long must it stand out at the rate of 6 per cent?

52) A gentleman puts out $2,000 at 4 per cent compound interest. In how many years shall he have drawn $1,400 interest from this sum?

Remark. If he shall have drawn $1,400 compound interest, his capital must have increased by $1,400. The question is therefore the same as this: in how many years will $1,000 have increased to $2,400 at the rate of 4 per cent compound interest?

53) I have a capital of $4,500. In how many years shall I be able to draw from it 10,000 interest, supposing compound interest is calculated at the rate of 6 per cent?

54) In how many years will the population of a place become 10 times as great as it is at present, if the yearly increase amount to 3 persons in a hundred?

Remark. This problem is analogous to those of numbers 45, 46, 47, 49, 50.

55) What is the rate of compound interest of a capital which in 18 years has doubled itself?

56) What is the rate of compound interest of a capital which in 24 years has tripled itself?

57) How long must $3,600 remain at 5 per cent compound interest, in order to accrue to the same amount as $5,000 at 4 per cent in 12 years?

Remark. If in formula I, (page 186) we take N for the unknown quantity, then we have

$$N = \frac{\log c + n \log p - \log C}{\log P};$$

but in this case we have

$$\begin{aligned} \log c &= \log 5000, \\ n \log p &= 12 \log 1.04, \\ \log C &= \log 3600, \\ \log P &= \log 1.05; \end{aligned}$$

consequently

$$N = \frac{\log 5{,}000 + 12 \log 1.04 - \log 3{,}600}{\log 1.05} = \text{\&c.}$$

58) How long must $10,000 remain at 6 per cent compound interest, in order to accrue as much as $8,000 at 8 per cent compound interest in 10 years?

59) How large must a capital C be, that at 4 per cent compound interest it may in 10 years hence amount to as much as $4,000 at 5 per cent compound interest, in 6 years?

Remark. If in formula I, (page 186) we take C for the unknown quantity, we have

$$\log C = \log c + n \log p - N \log P.$$

But we have $N \log P = 10 \log 1.04,$

$$\begin{aligned} \log c &= \log 4000, \\ n \log p &= 6 \log 1.05, \end{aligned}$$

hence $\log C = \log 4{,}000 + 6 \log 1.05 - 10 \log 1.04,$ whence, &c.

60) What is the amount of a capital which at 5 per cent compound interest becomes in 8 years as large as a capital of $15,000 at 6 per cent compound interest in 12 years?

61) What must be the rate of compound interest, in order that a capital of $3,000 shall in 6 years amount to as much as a capital of $4,000 in 5 years at 7 per cent interest?